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UNIVERSITY OF WISCONSIN–MADISON
Celebrating 60 Years

Compiling Quantum Circuits

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ASPLOS 2025 Tutorial

Outline

Part I: Quantum Computing Fundamentals

A. Qubits, gates, and circuits

B. Quantum algorithms

C. Quantum Hardware & Error-correction

Part II: Quantum-Circuit Optimization

A. Rewrite rules

B. Circuit resynthesis

C. Scheduling

Part III: Qubit Mapping and Routing

A. QMR for near-term devices

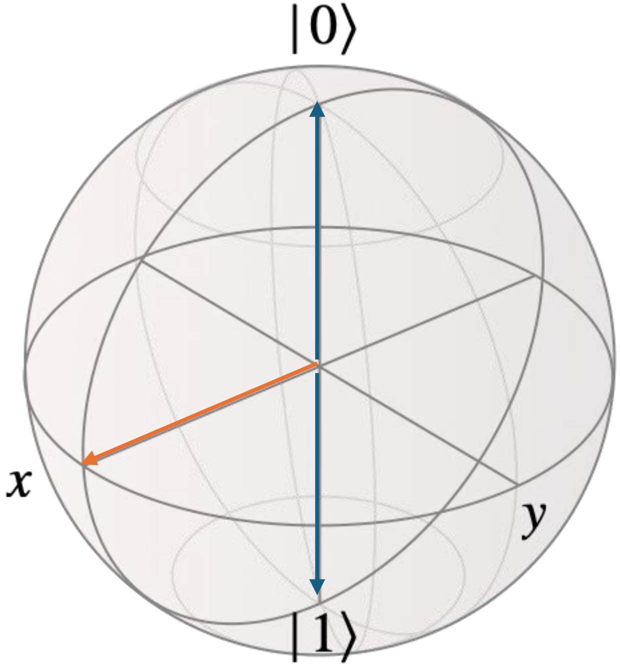
B. QMR for fault-tolerant devices

Part IV: A tour of `wisq`

Quantum Computing Fundamentals

Bits and **qu**bits

Classical bit



Qubit

Qubit states

Position on the surface of a sphere is a 2D vector

We write these vectors like this:

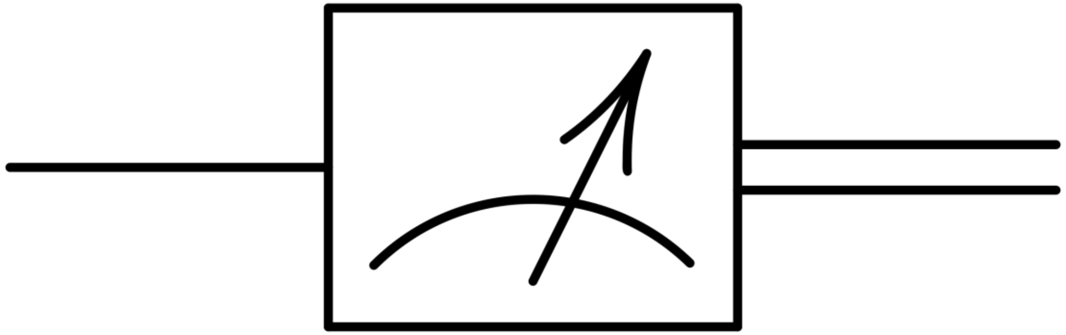
Basis vectors

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Complex numbers

Measurement

Quantum mechanics forbids direct access to α and β



Implication: $|a|^2 + |\beta|^2 = 1$

The X gate

Gates transform the state of a qubit

Classical NOT: 0 → 1, 1→ 0
Quantum NOT: $\alpha|0\rangle + \beta|1\rangle \rightarrow \beta|0\rangle + \alpha|1\rangle$

In other words,

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

The Hadamard gate

Produces equal *superposition* of states

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$\mathrm{H}|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\mathrm{H}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

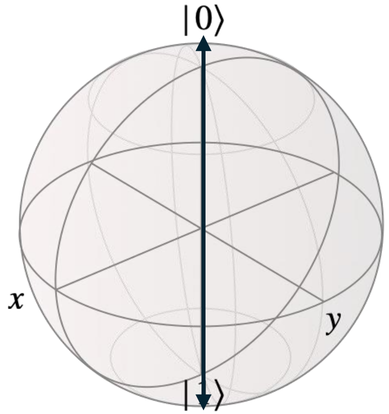
Single qubit gates

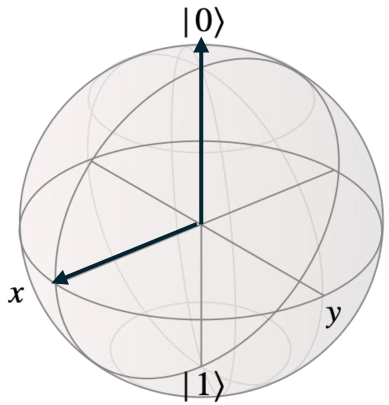
2x2 (unitary) matrices

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Rotations on the Bloch Sphere





Multi-qubit systems

Two qubit state:

$$\alpha_1|00\rangle + \alpha_2|01\rangle + \alpha_3|10\rangle + \alpha_4|11\rangle$$

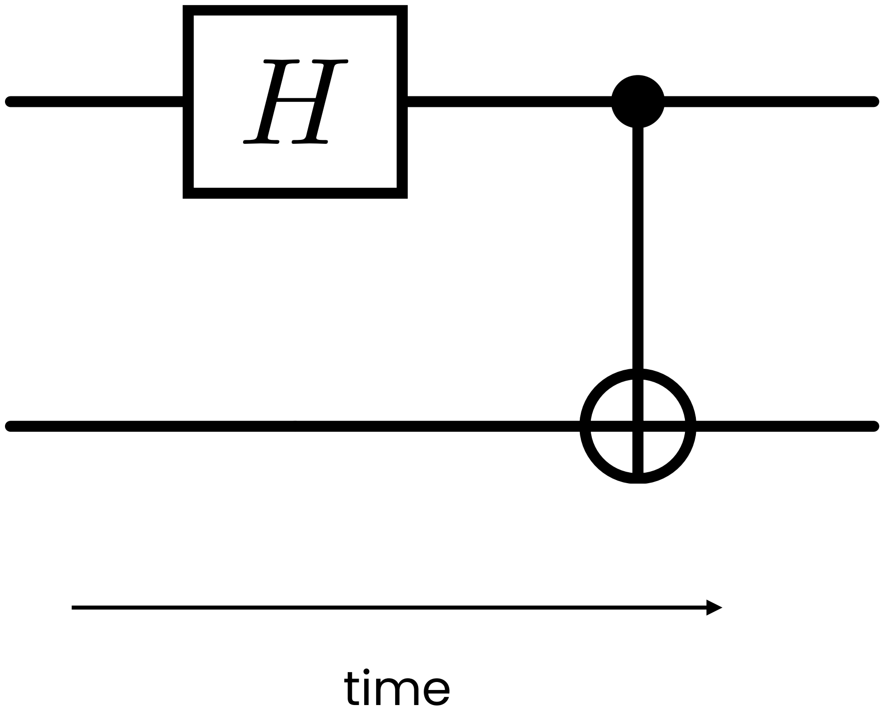
Controlled-NOT (CNOT gate):

“Quantum XOR”

$|xy\rangle \mapsto |x(x \oplus y)\rangle$

Quantum circuits

```
H q0;
CNOT q0, q1;
```



Quantum Algorithms

Next: two examples of computing with quantum circuits

- Deutsch-Jozsa (first exponential speedup found)
- Grover’s search algorithm

Just for motivation: algorithmic details aren’t crucial

Won’t get cover other important applications

- Shor’s Algorithm
- Hamiltonian Simulation

Deutsch-Jozsa Problem

Given oracle access to a function $f : \{0,1\}^n \rightarrow \{0,1\}$

Promised that f is either *constant* or *balanced*

e.g. $f(x_1 \dots x_n) = 1$

e.g. $f(x_1 \dots x_n) = x_1$

How many queries do we need to figure out which?

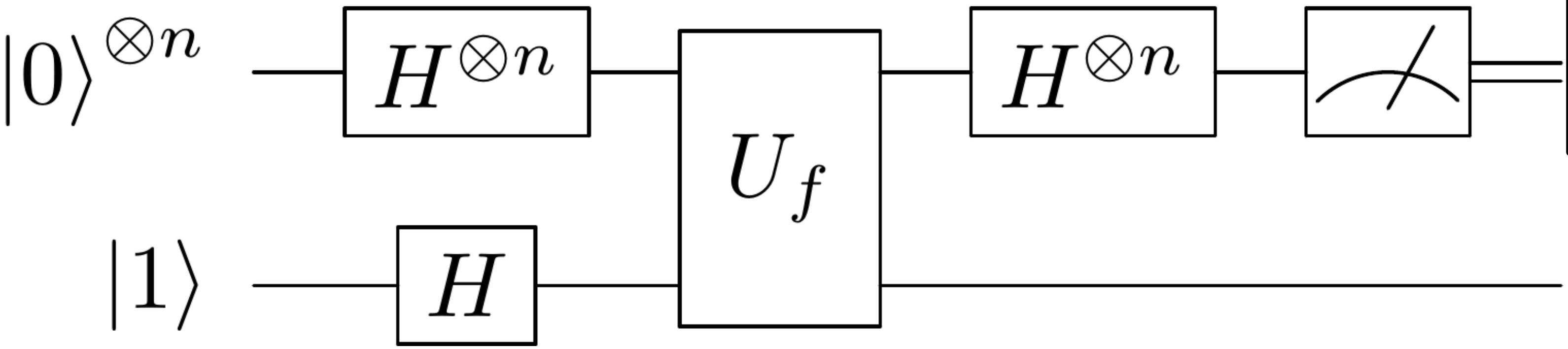
Classically: $2^{n-1} + 1$

Quantum: 1

Deutsch-Jozsa Circuit

Quantum operations must be unitary, so we use a reversible oracle

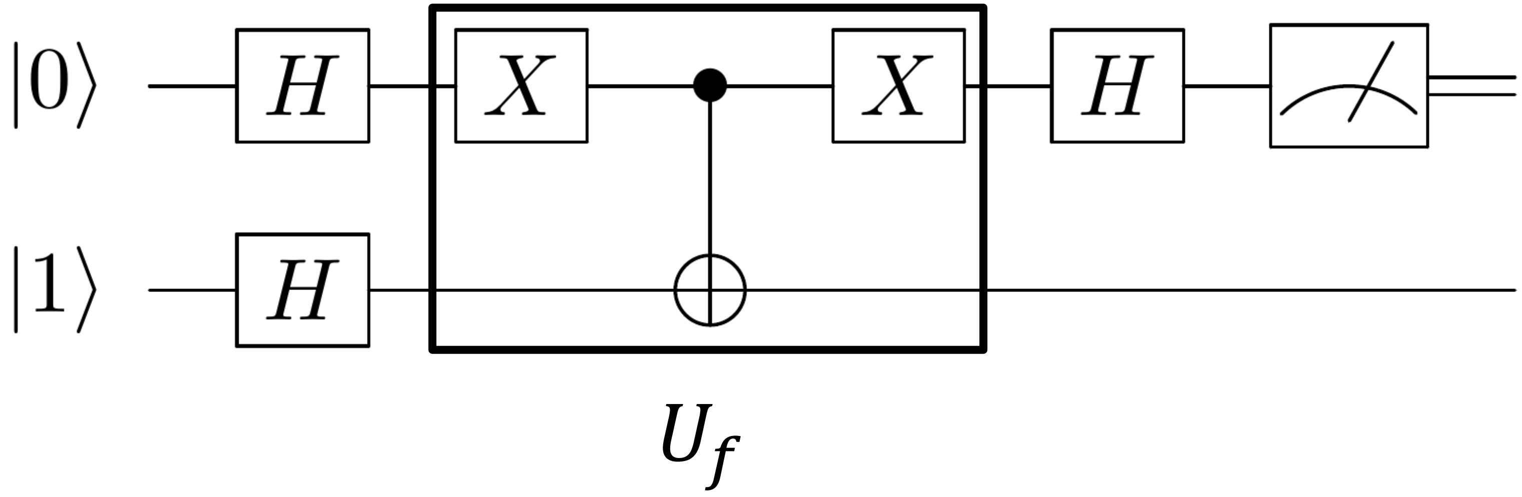
$$\mathrm{U_f}(|xy\rangle) = |x(y \oplus f(x))\rangle$$



<u>Prob of measuring all 0</u>
1 if constant
0 if balanced

Deutsch-Jozsa w/ Concrete Oracle

Here's the circuit where f is the "NOT" function



Grover's Problem

O($\sqrt{N}$) Quantum speedup for unstructured search

Again, have oracle access to a function $f : \{0,1\}^n \to \{0,1\}$

Looking for the element(s) x out of $N = 2^n$ choices such that $f(x) = 1$.

How many queries do we need?

Classically: *N*

Quantum: $\approx \frac{\pi}{4}\sqrt{N}$

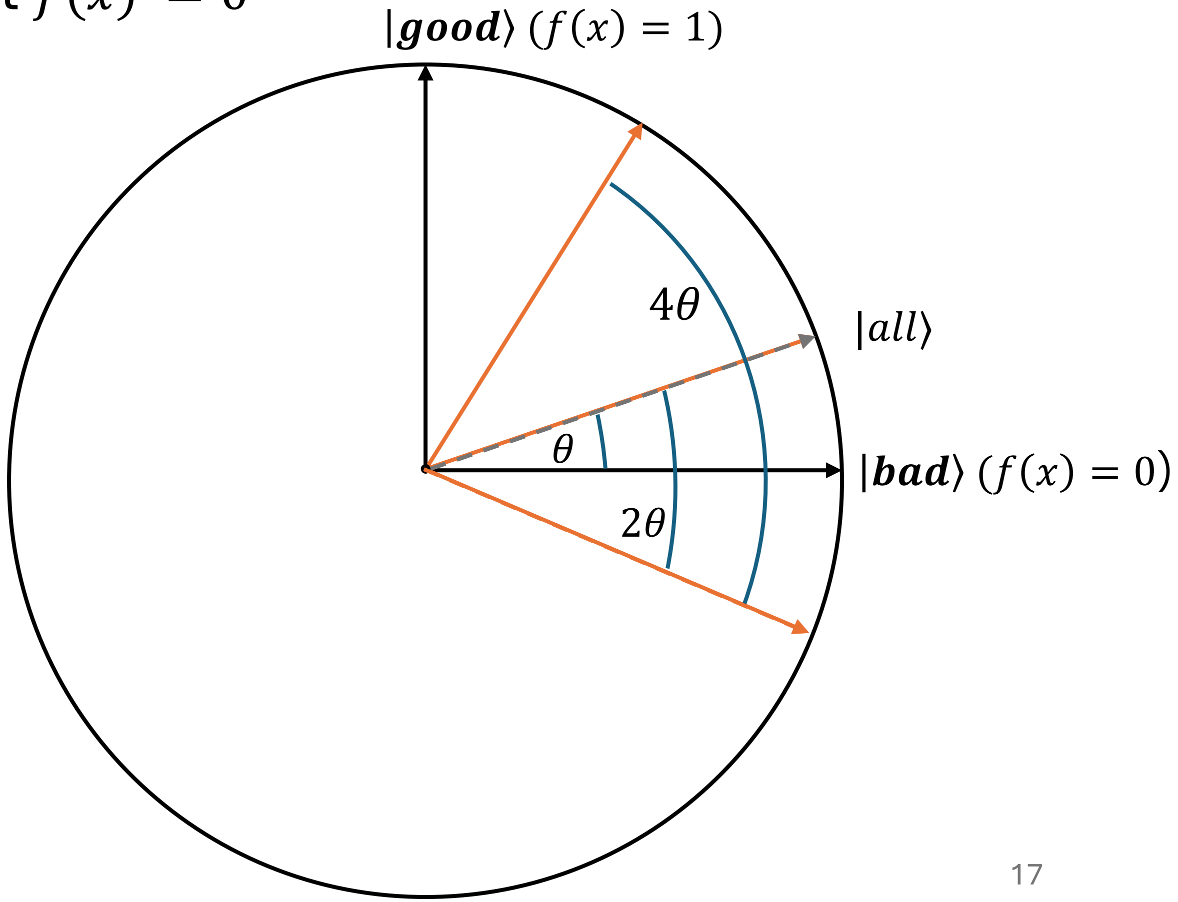
Geometric intuition for Grover's

Consider the plane spanned by two vectors:
$|good\rangle$: equal superposition over x such that $f(x) = 1$
$|bad\rangle$: equal superposition over x such that $f(x) = 0$

<u>Grover iteration</u>

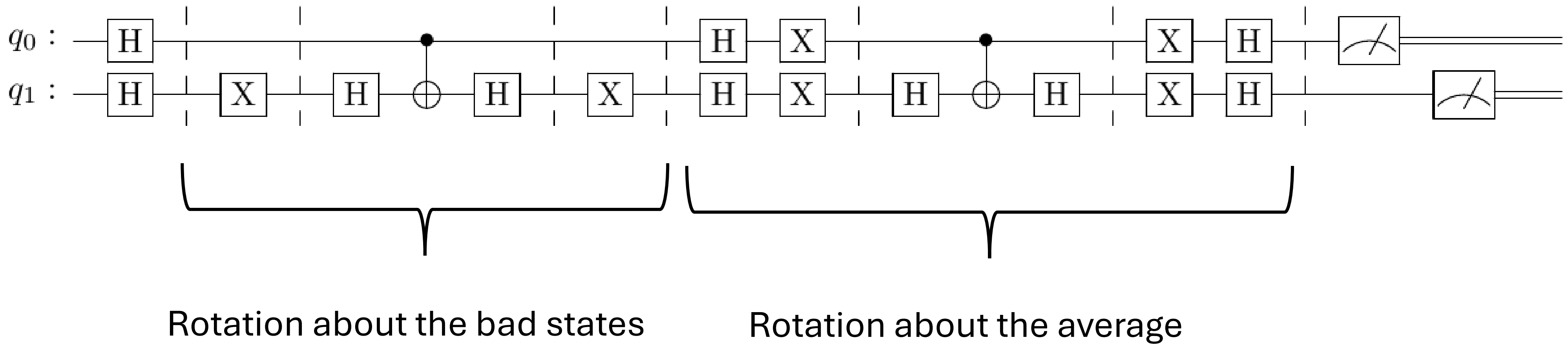
1. Rotate about bad states
2. Rotate about all states

Result: get closer to the good axis



Grover Circuit

An example single-iteration circuit where our target state is $|10\rangle$



The NISQ era

***N**oisy **I**ntermediate-**S**cale **Q**uantum*

Lack resources for error correction (instead rely on error *mitigation*)
Sample many runs due to low probability of error-free outcome

Most promising applications find approximate solutions:

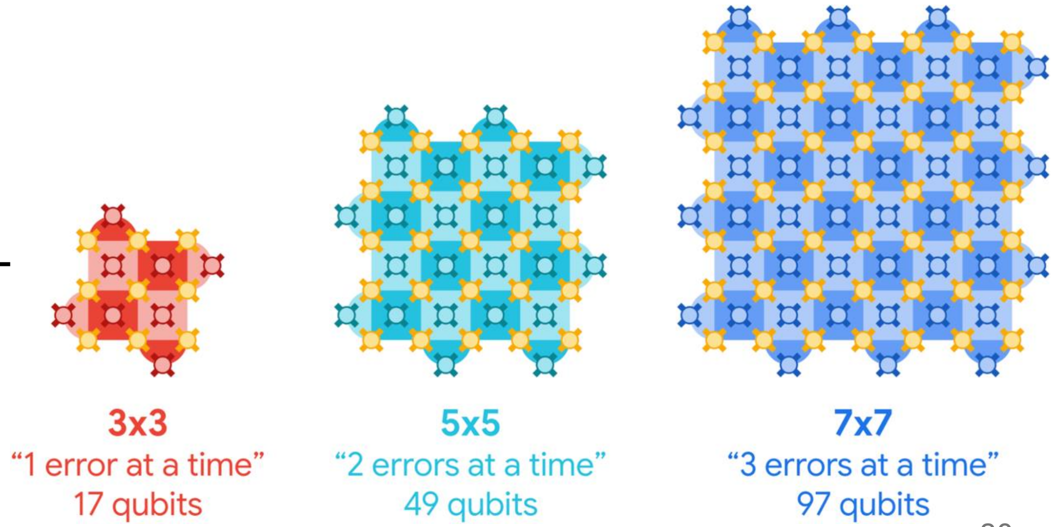
- Quantum Approximate Optimization Algorithm
- Variational Quantum Eigensolver

Quantum Error Correction

Encode a logical qubit into several physical qubits
Reduce error by scaling the logical qubit

Prerequisite for exciting applications like Shor's, Quantum simulation

Most promising approach
Surface Codes



https://research.google/blog/making-quantum-error-correction-work/

Surface codes in hardware

Article | Published: 25 May 2022

Realizing repeated quantum error correction in a distance-three surface code

Realization of an Error-Correcting Surface Code with Superconducting Qubits

Youwei Zhao[1,2,3,*], Yangsen Ye[1,2,3,*], He-Liang Huang[1,2,3,*], Yiming Zhang[1,2,3], Dachao Wu[1,2,3], Huijie Guan[1,2,3], Qingling Zhu[1,2,3], Zuolin Wei[1,2,3], Tan He[1,2,3] *et al.*

Show more

Phys. Rev. Lett. **129**, 03

DOI: https://doi.org/10

Article | Published: 09 December 2024

Quantum error correction below the surface code threshold

Google Quantum AI and Collaborators

Nature (2024) | Cite this article

The Quantum Software Stack