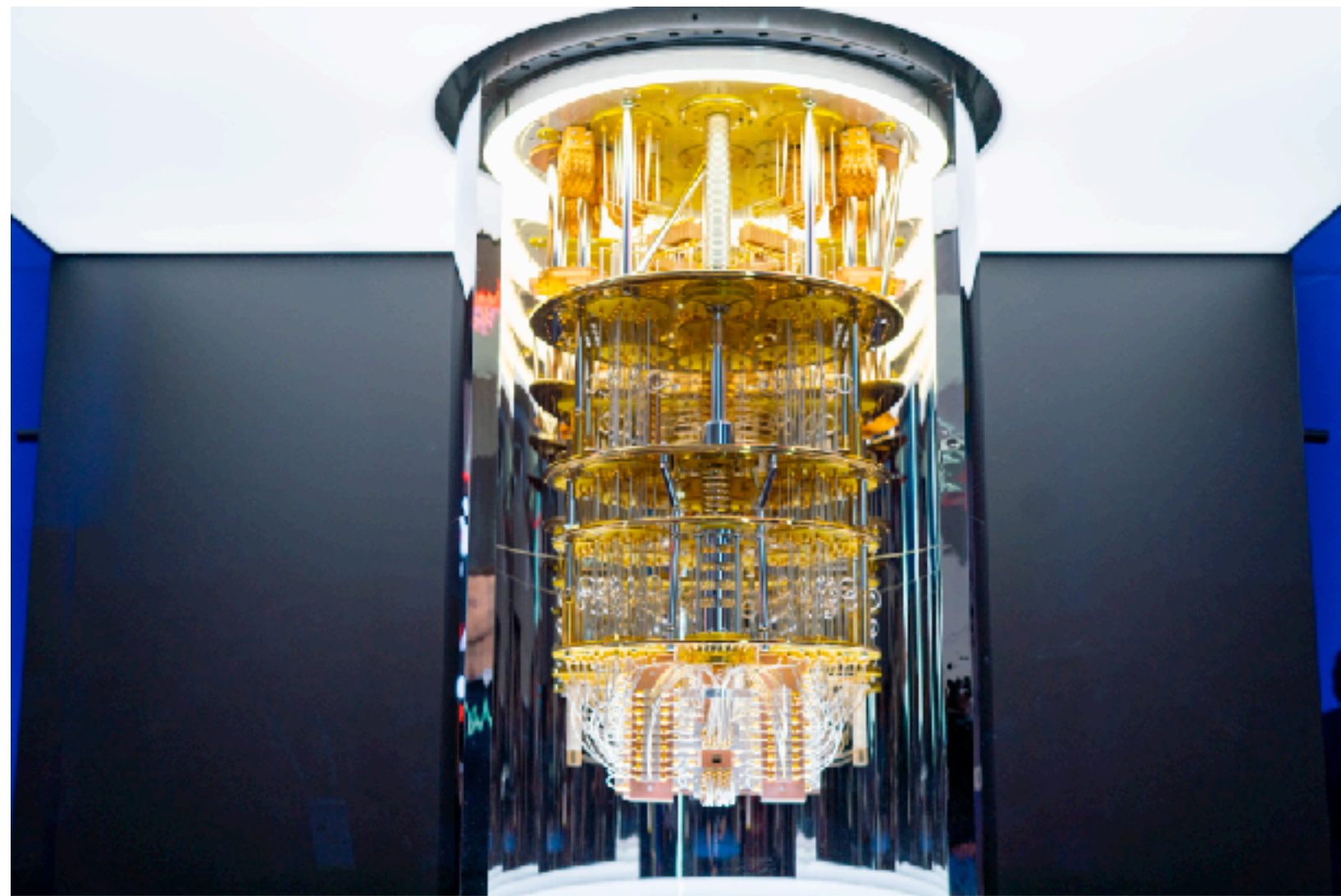
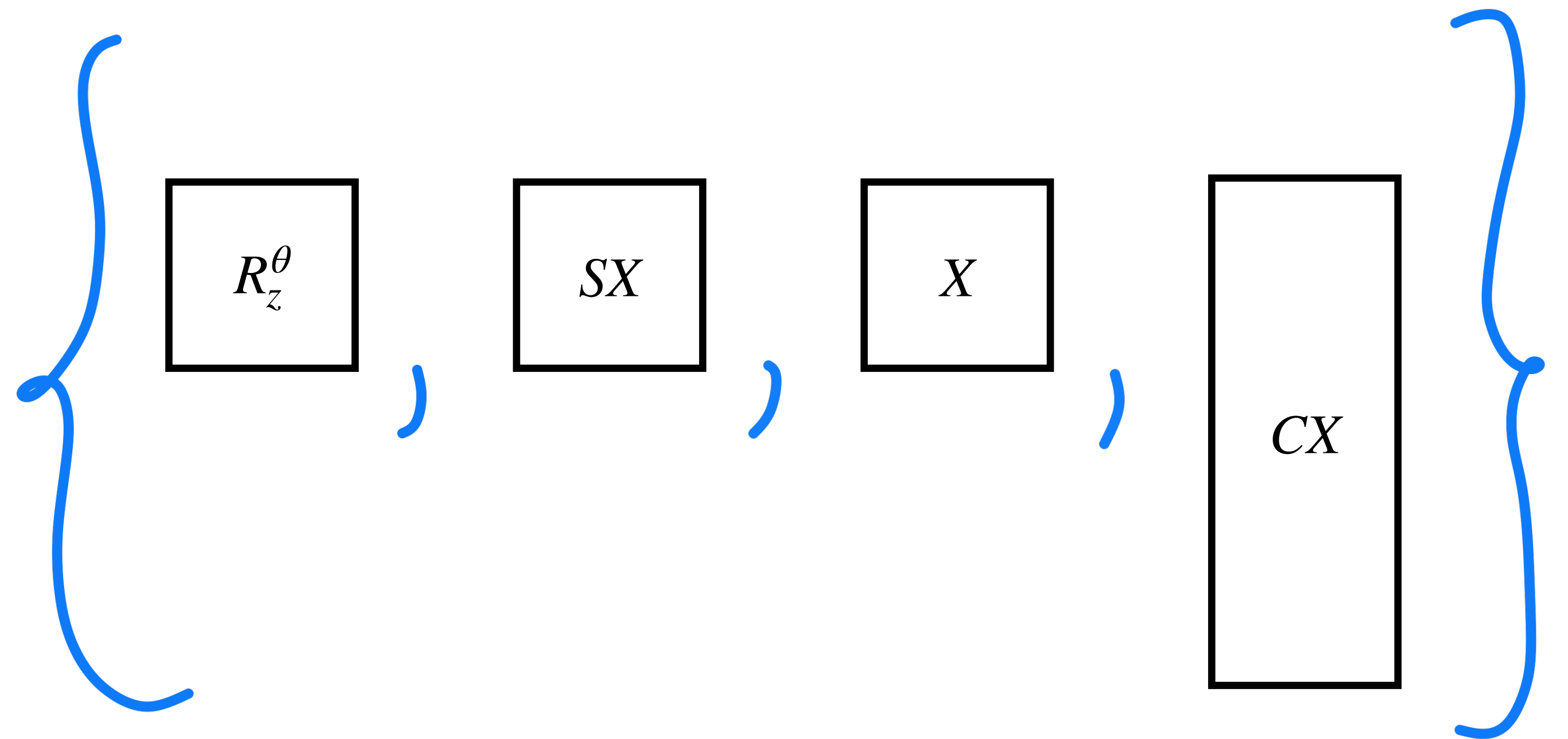


Quantum-Circuit Optimization

Native Basis Gate Set



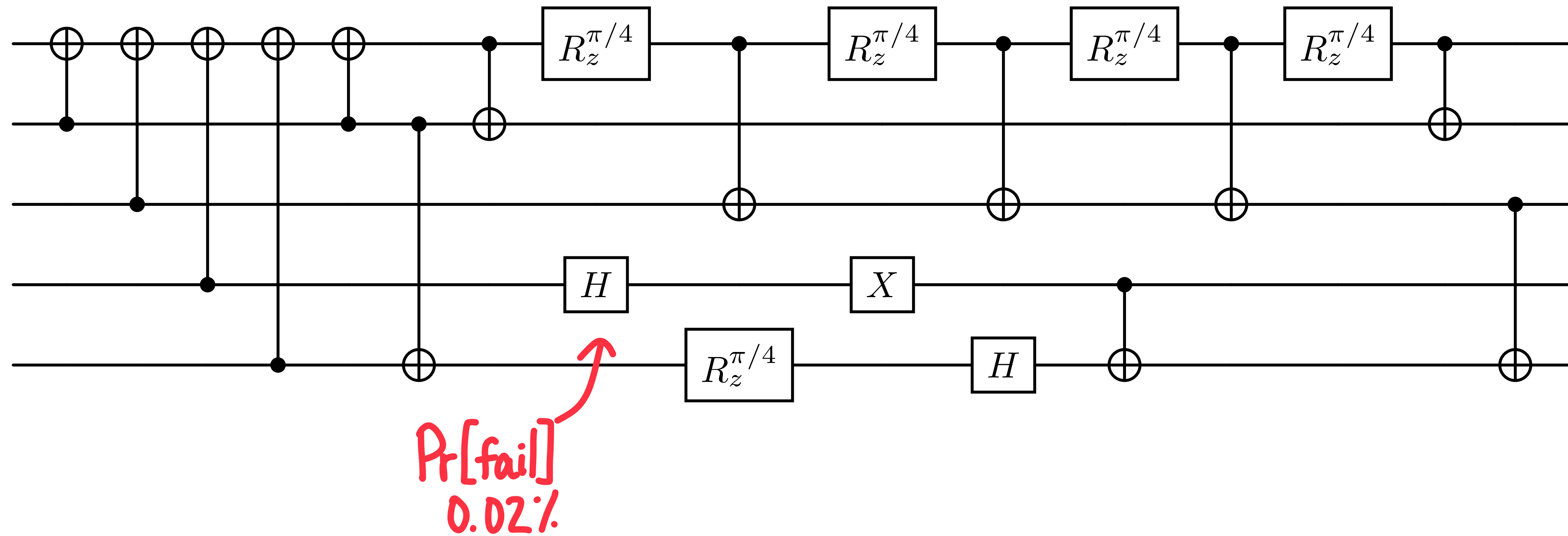
IBM



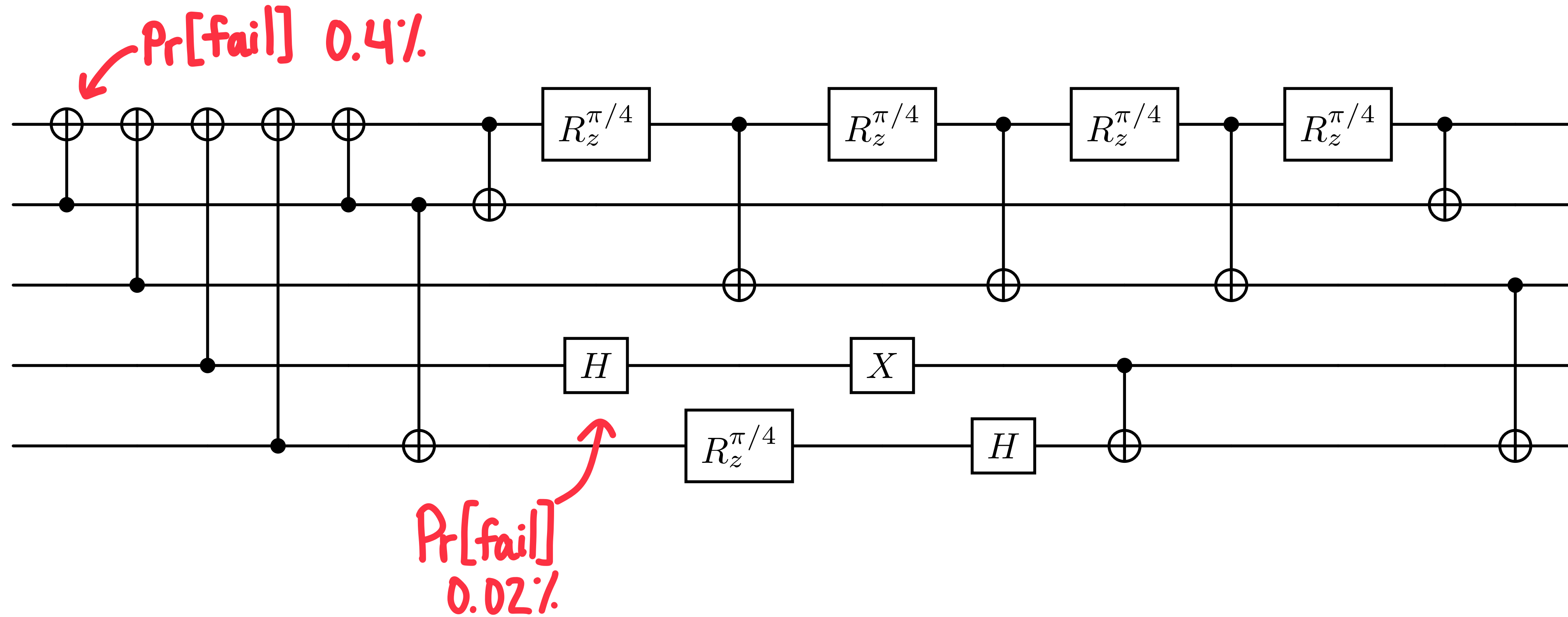
"universal"

i.e. approximate any computation to arbitrary precision

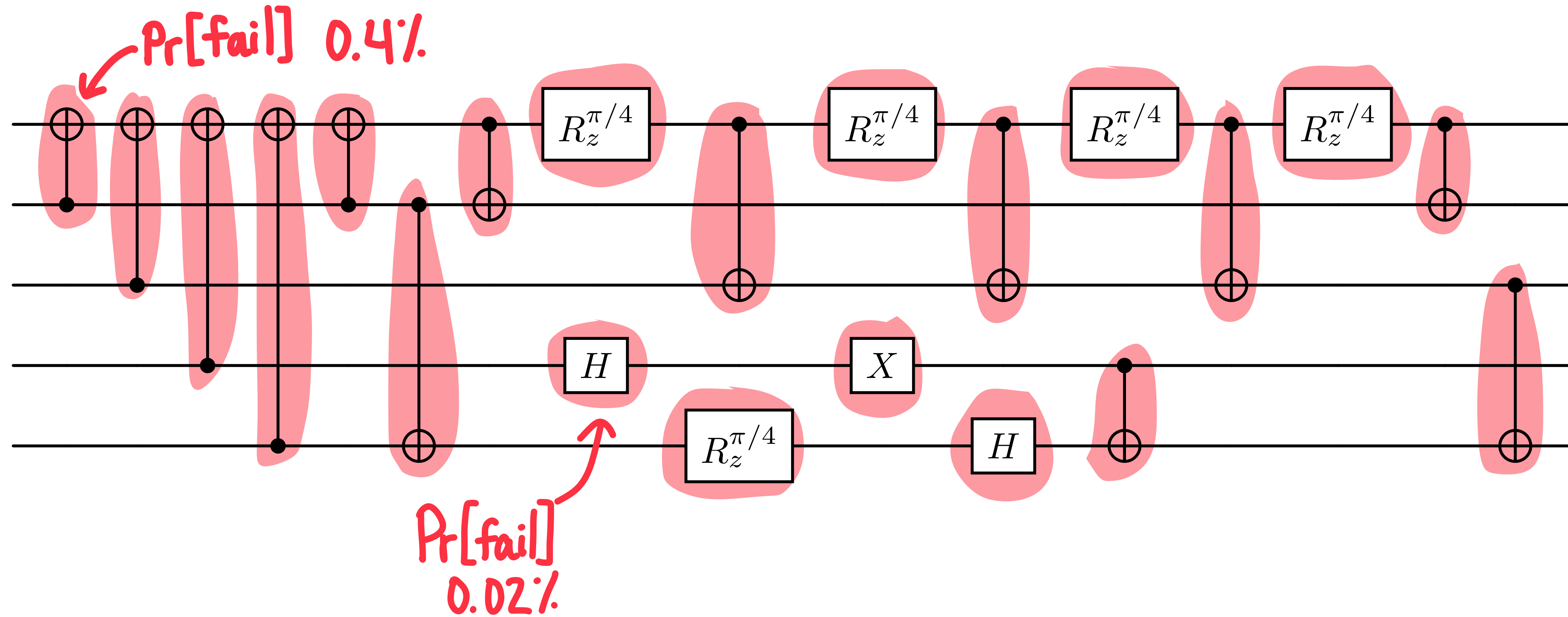
Optimization Objectives



Optimization Objectives



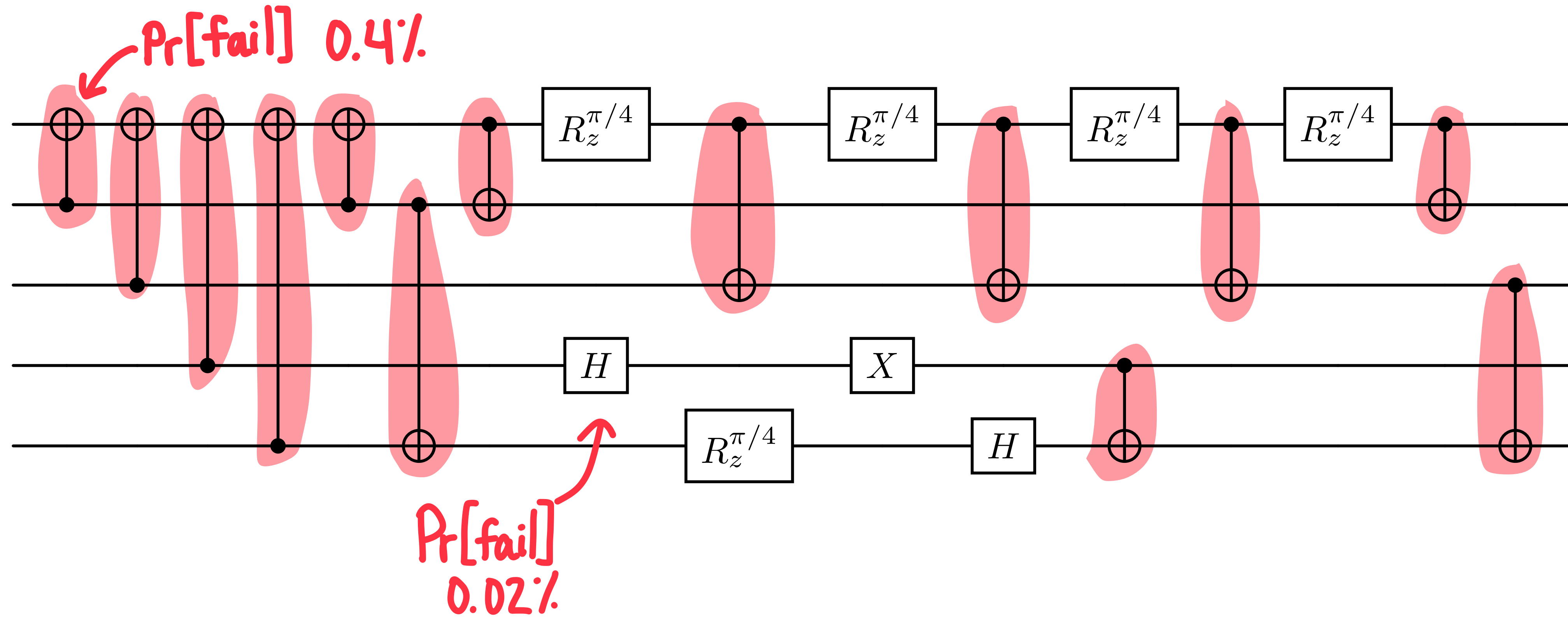
Optimization Objectives



NISQ

- total gate count

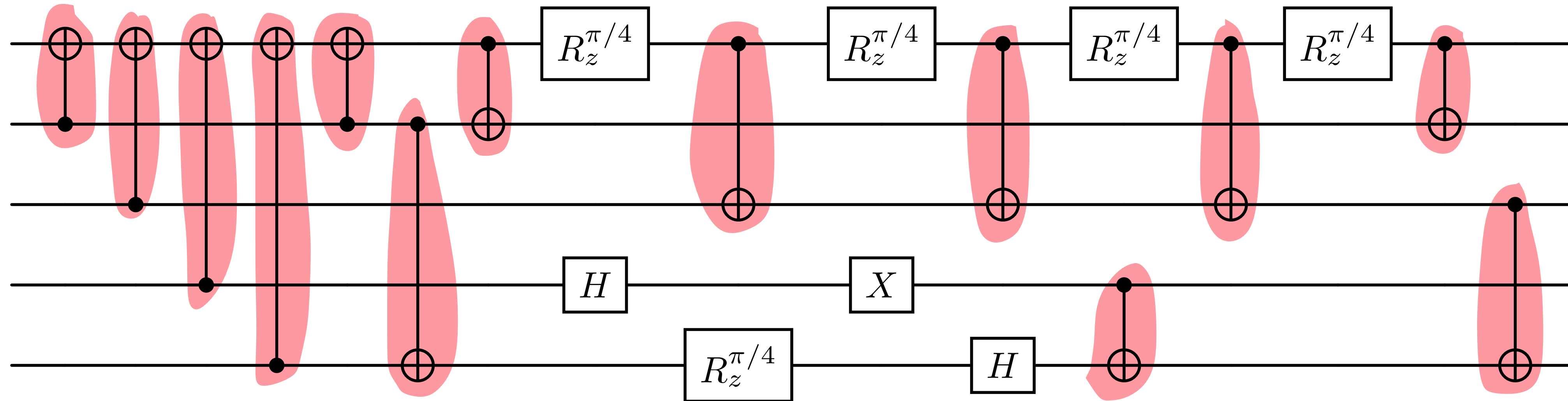
Optimization Objectives



NISQ

- total gate count
- 2q gate count

Optimization Objectives



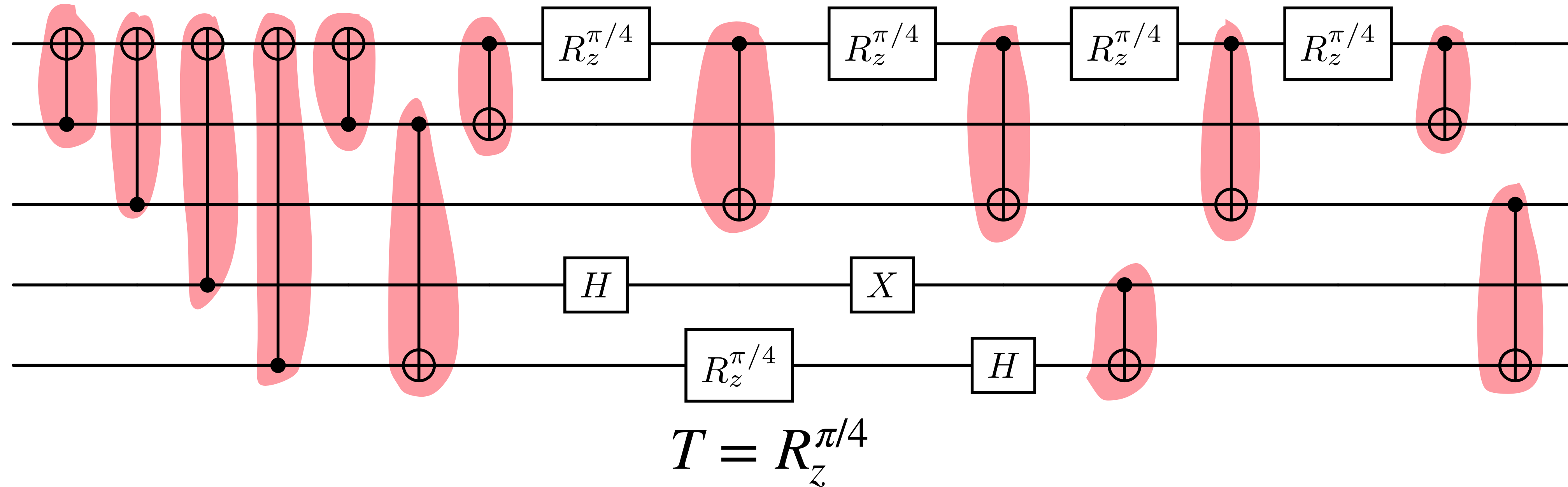
NISQ

- total gate count
- 2q gate count

FTQC

- T count and 2q gate count

Optimization Objectives



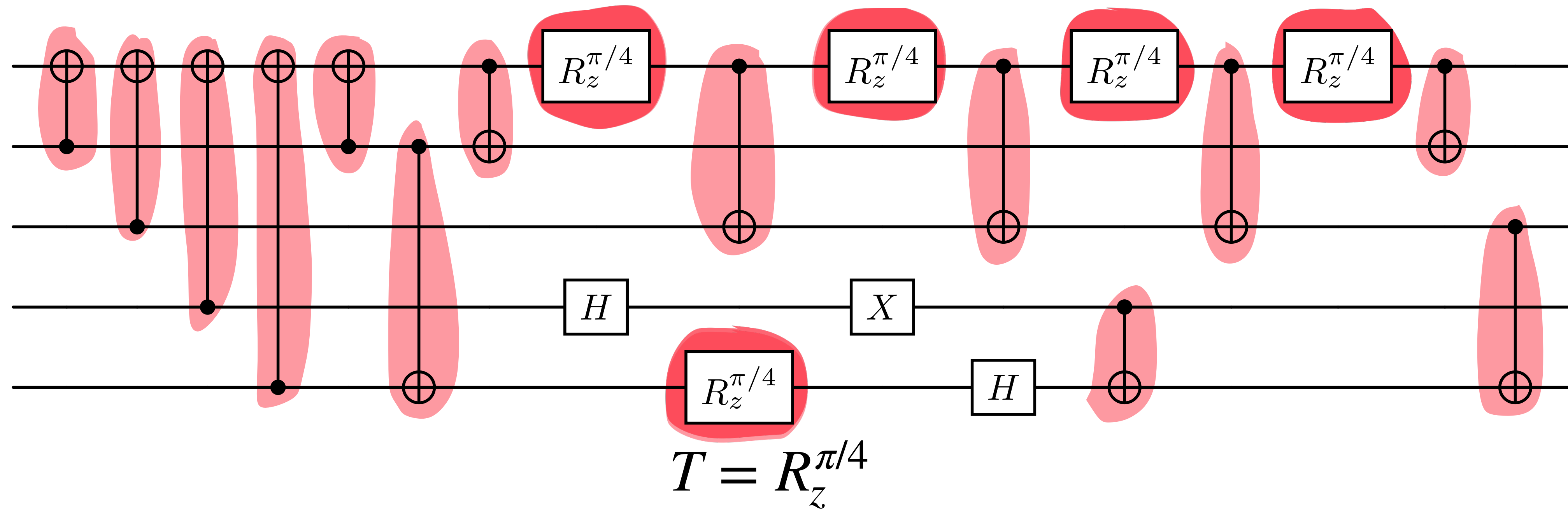
NISQ

- total gate count
- 2q gate count

FTQC

- T count and 2q gate count

Optimization Objectives



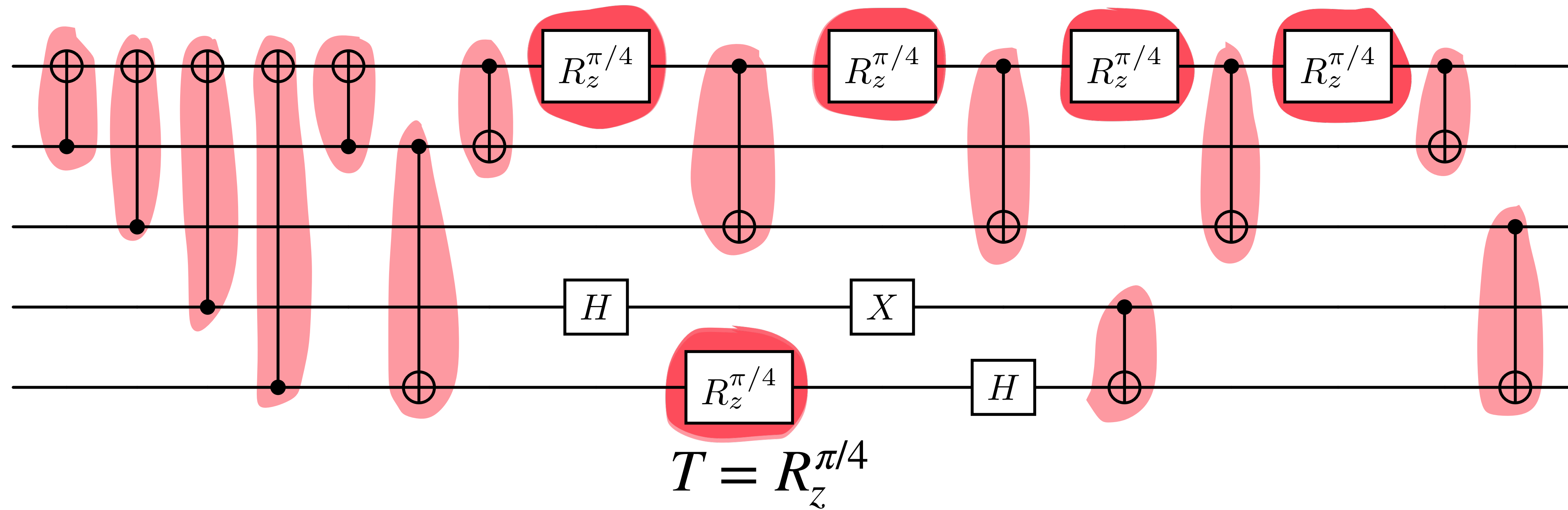
NISQ

- total gate count
- 2q gate count

FTQC

- T count and 2q gate count

Optimization Objectives



NISQ

- total gate count
- 2q gate count

and many more...

FTQC

- T count and 2q gate count

High-level problem

Given a circuit and optimization objective, output an equivalent circuit that minimizes the optimization objective.

Optimising quantum circuits is generally hard

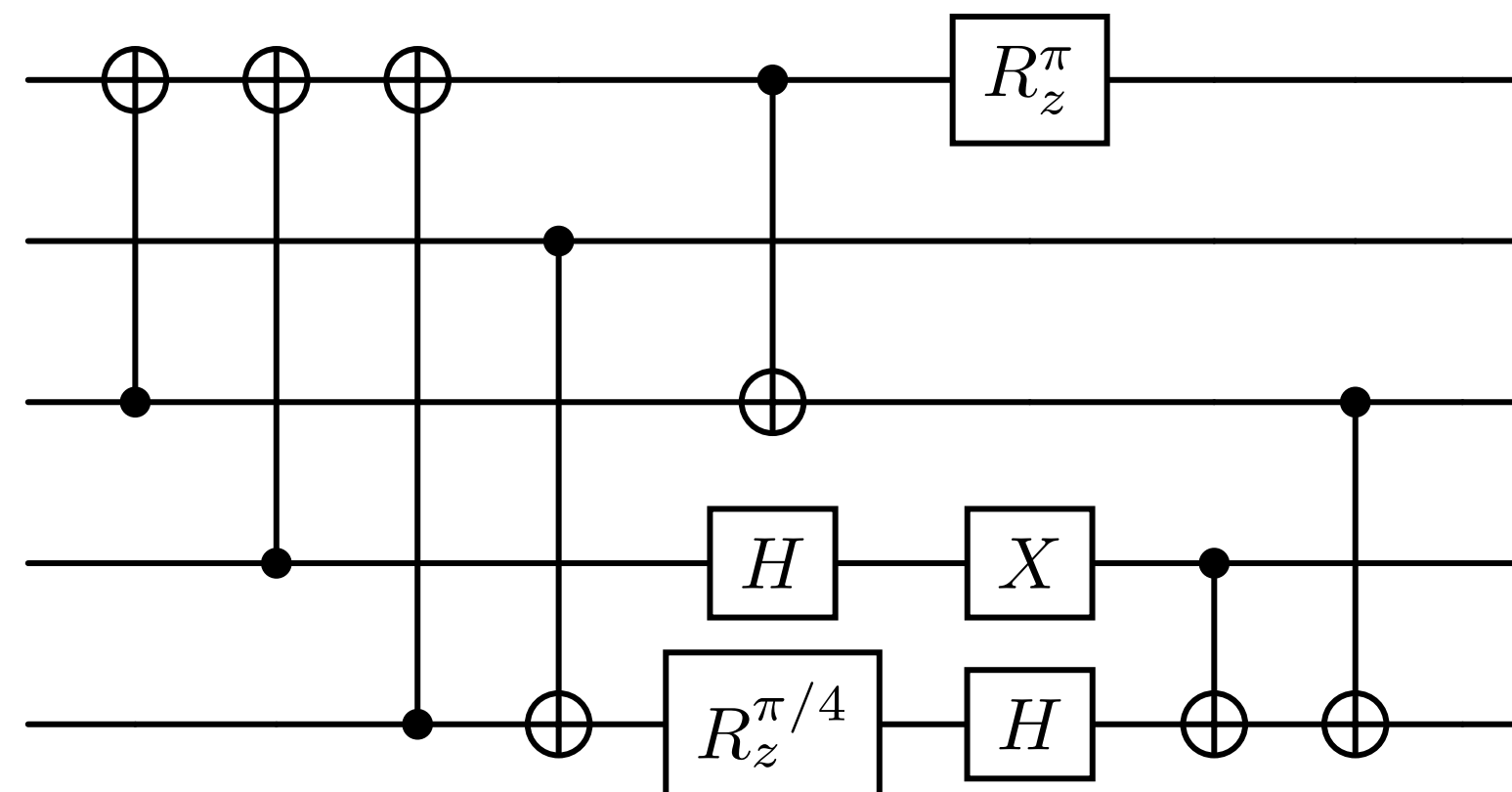
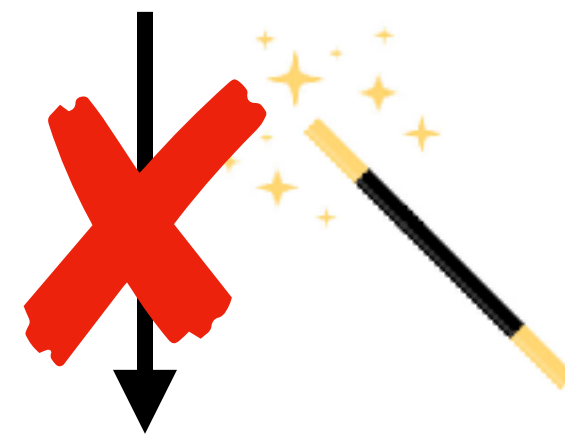
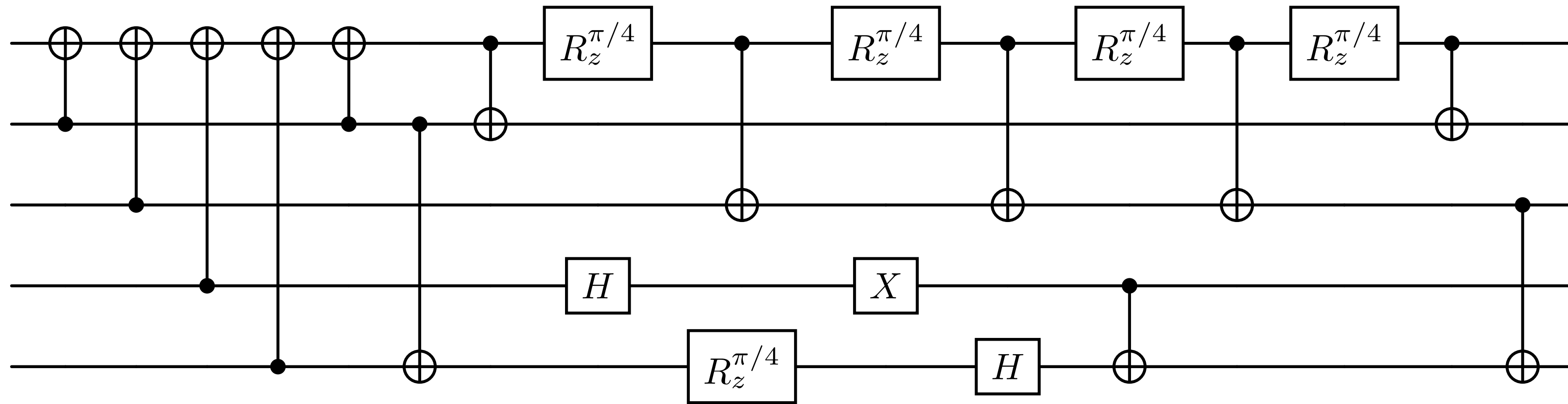
John van de Wetering¹ and Matthew Amy²

¹University of Amsterdam

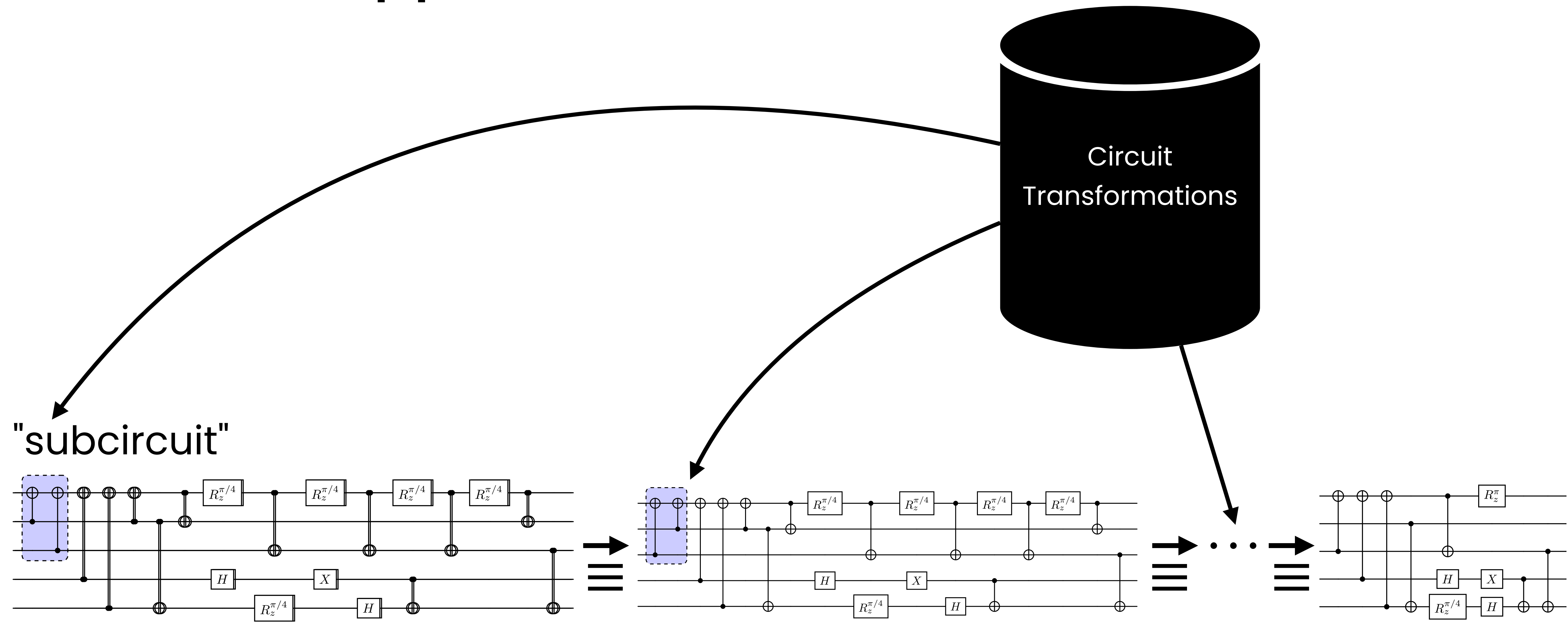
²Simon Fraser University

July 28th 2024

General Approach



General Approach



Circuit Equivalence

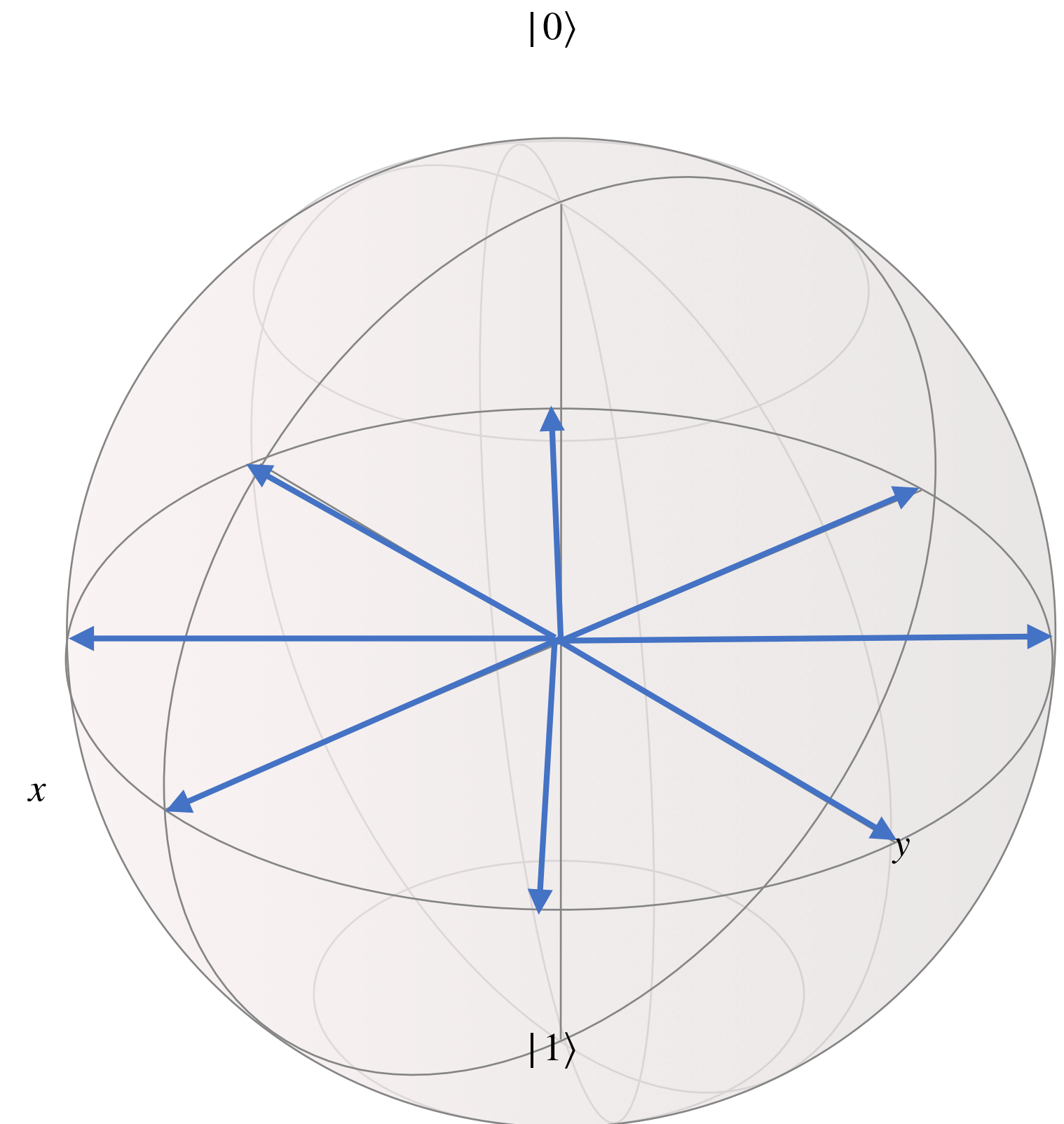
Circuits are unitary matrices: $U^\dagger U = U U^\dagger = I$

$$C_1 \equiv C_2 \iff U_{C_1} = U_{C_2}$$

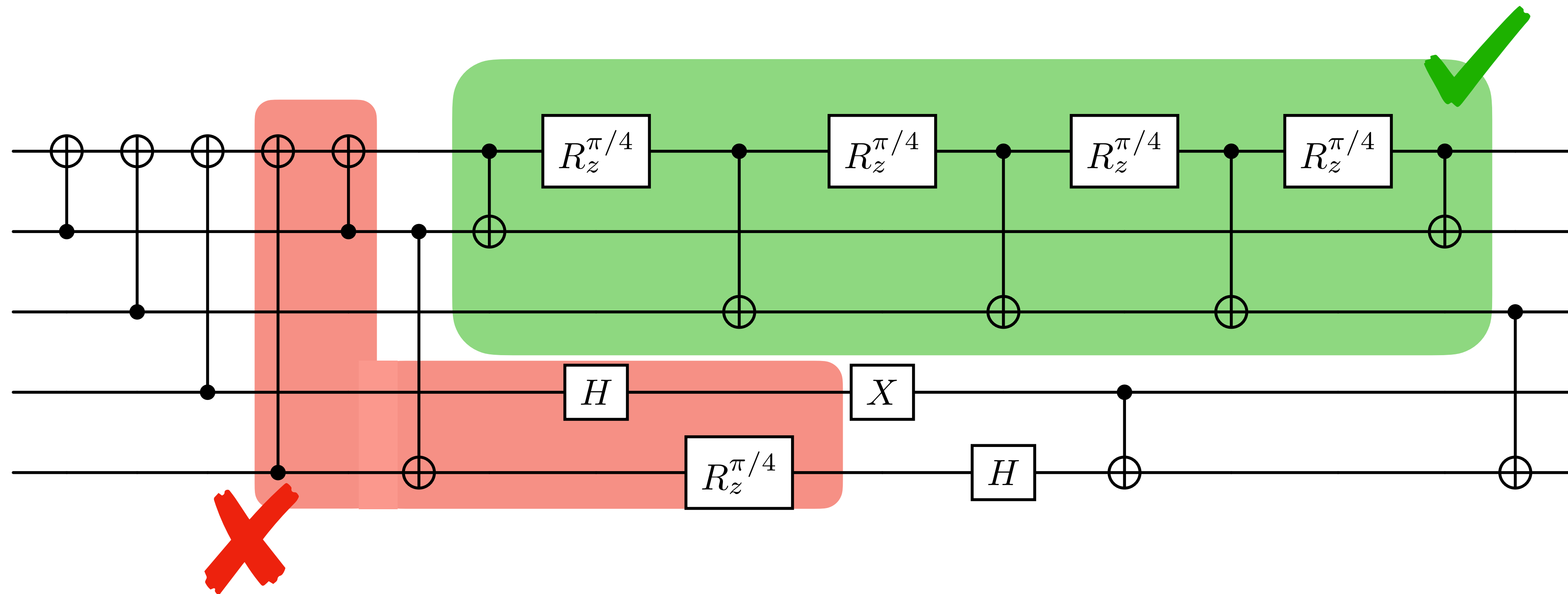
"global phase"

$$C_1 \equiv C_2 \iff \exists \varphi . U_{C_1} = e^{i\varphi} U_{C_2}$$

Hard for even a quantum computer to verify
(QMA-Complete)



Subcircuit



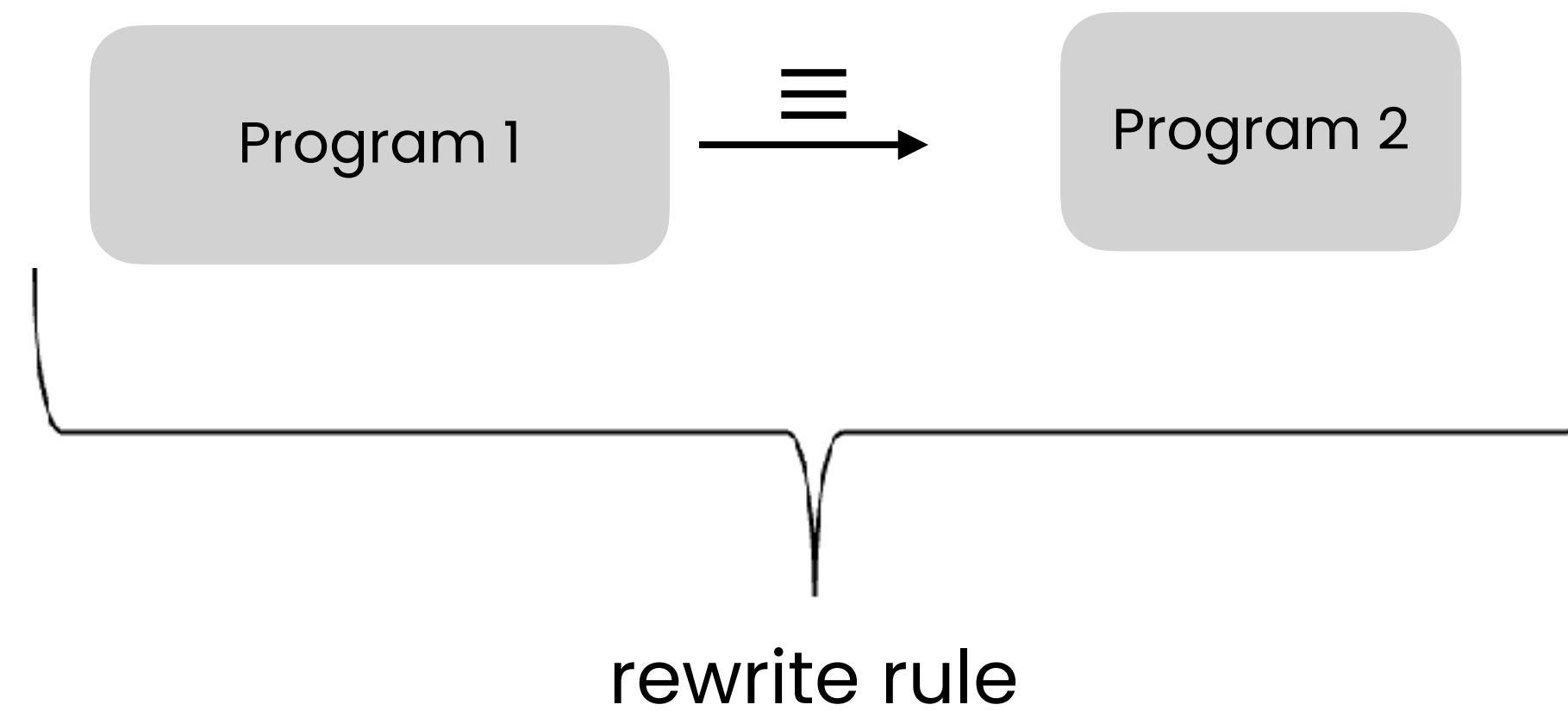
"convex subgraph"

i.e. any path between two vertices in the DAG also in subgraph

Rewrite Rules

Rewrite Rules

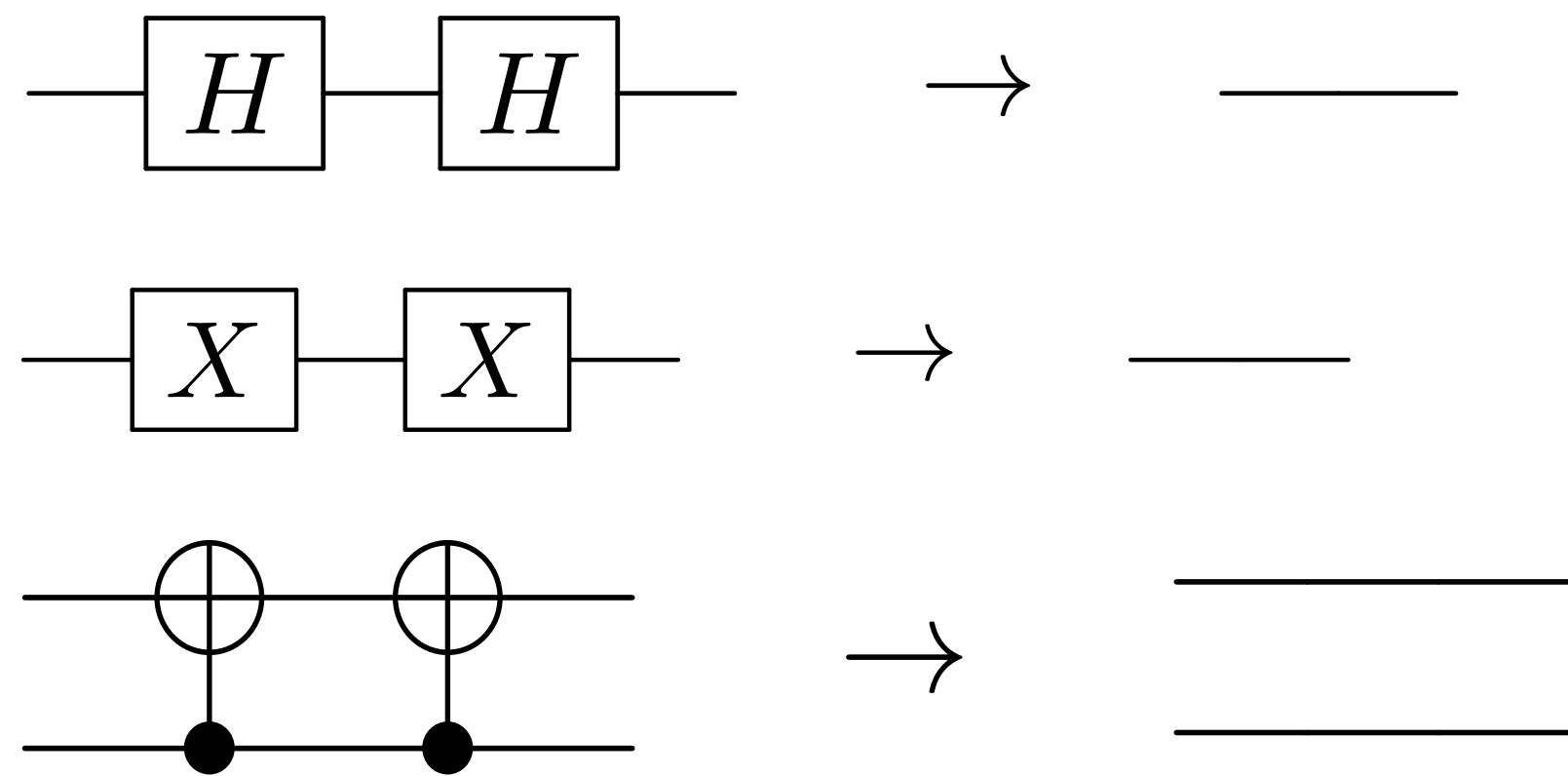
"peephole optimization"



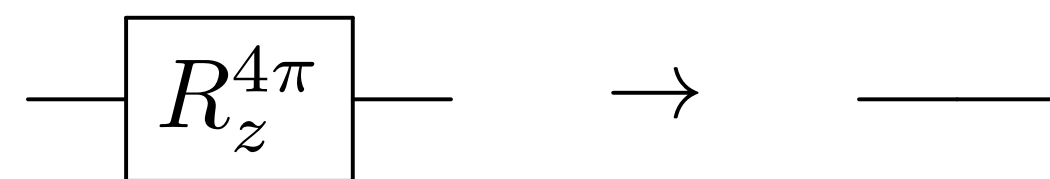
- Optimization: LHS and RHS same gate set
- Decomposition: LHS higher level than RHS

Simple Rewrite Rules

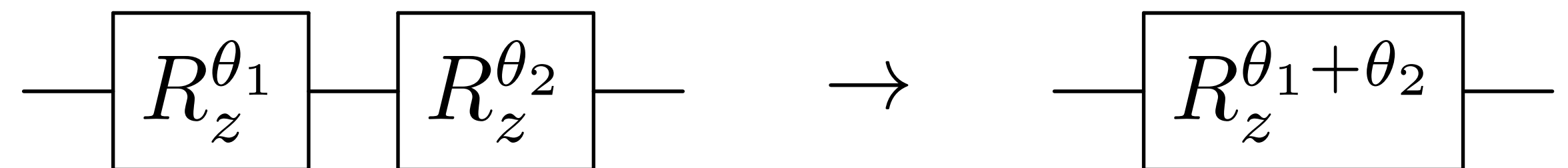
Cancel



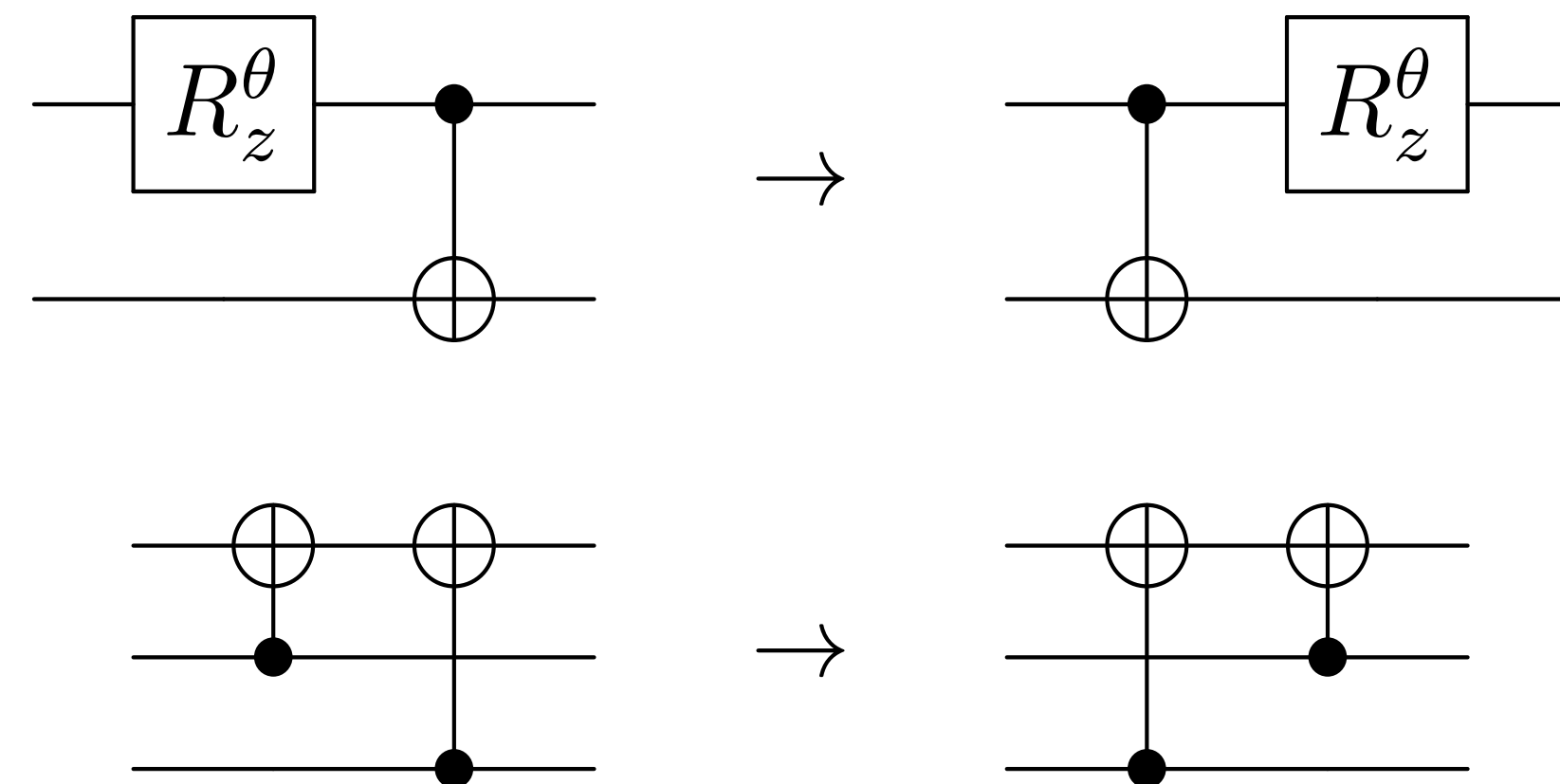
Global Phase



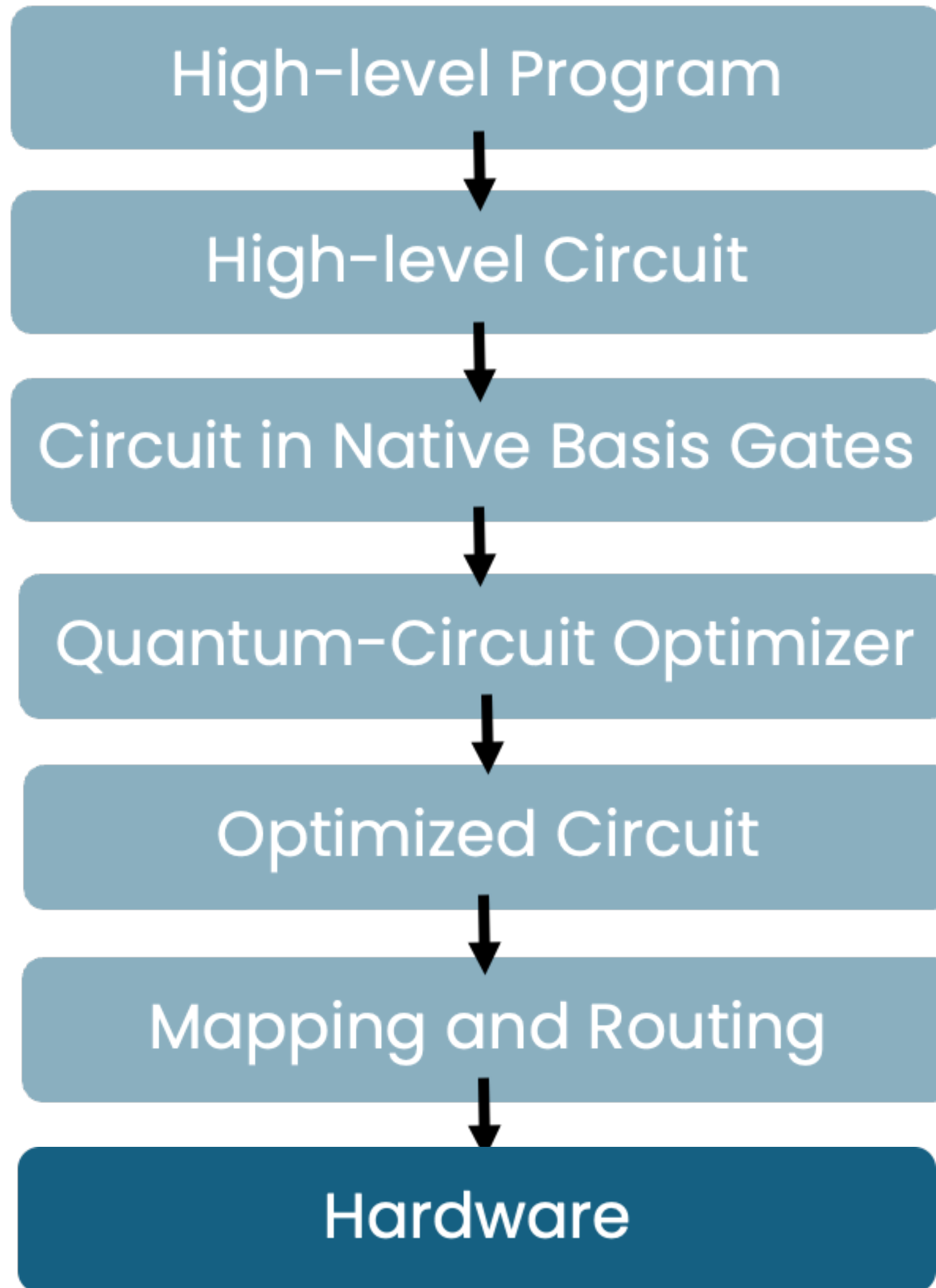
Merge



Commute



Diverse Hardware

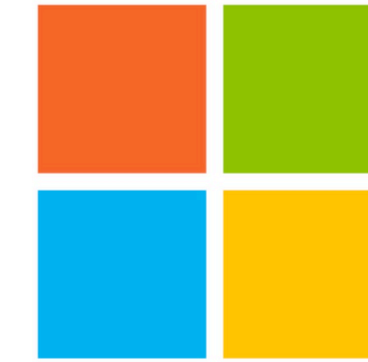


Diverse Hardware

Ion Trap

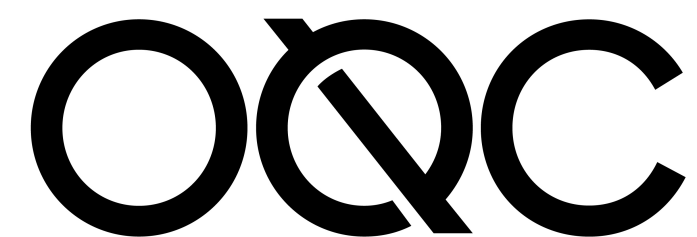


Topological



Hardware

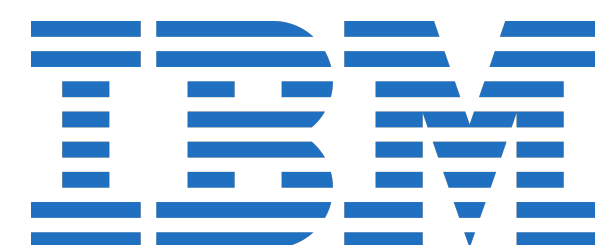
Superconducting



Neutral Atom



Google
Quantum AI



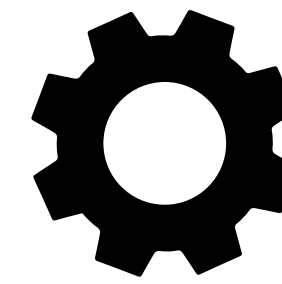
~~U1, U2, U3, CX~~ → ~~Rz, SX, X, CX~~



→ Rz, Rx, SX, X, CZ, Rzz

Synthesis!

Arbitrary gate set



Rewrite Rules

Naïve Synthesis

```
rules = []
circuits = enumerate(max_qubits, max_size)
for c1 in circuits:
    for c2 in circuits:
        if verify(c1, c2):
            rules.append(c1 → c2)
```

big (arrow pointing to `circuits`)

uh oh... (bracketed next to the nested loops)

expensive (arrow pointing to `verify`)

QUESO

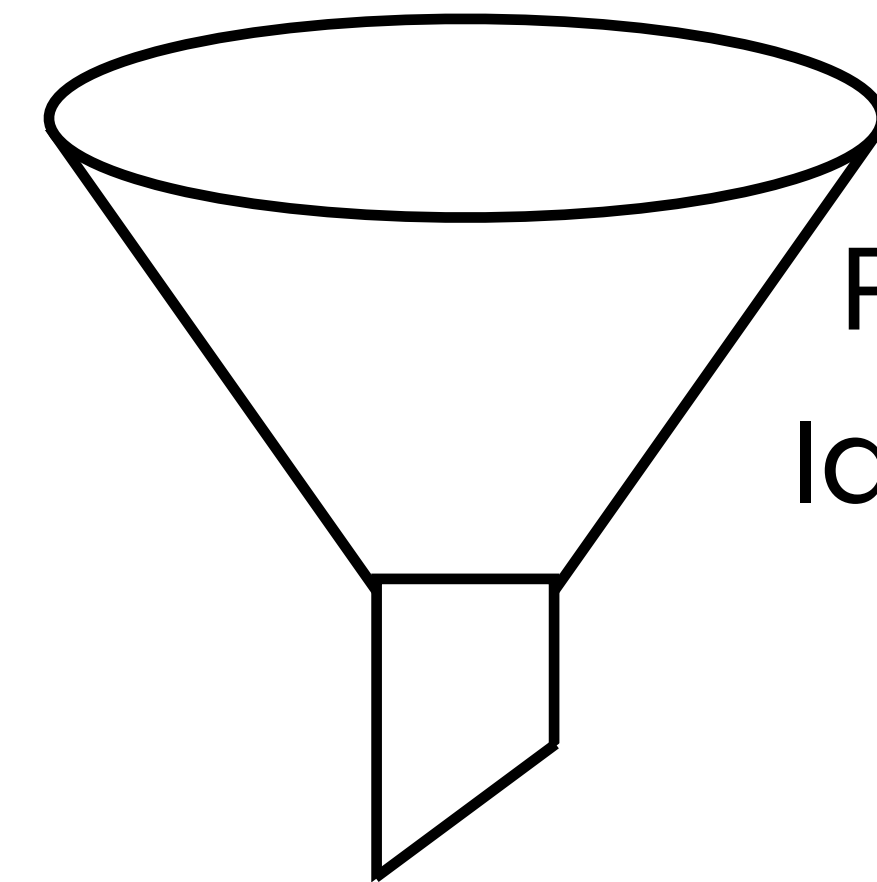
Schwartz-Zippel Lemma for polynomial identity testing (PIT)

Feynman's path integral formulation for quantum mechanics

(Symbolic) Circuit



Polynomial



Polynomial Identity Filter (PIF)



Circuit Equivalence Classes

Synthesizing Quantum-Circuit Optimizers

AMANDA XU, University of Wisconsin-Madison, USA

ABTIN MOLAVI, University of Wisconsin-Madison, USA

LAUREN PICK, University of Wisconsin-Madison, USA

SWAMIT TANNU, University of Wisconsin-Madison, USA

AWS ALBARGHOUTH, University of Wisconsin-Madison, USA

@ PLDI'23

Path-sum Semantics

Algebraically represent semantics of quantum gates

$$R_z(\theta) : |x\rangle \rightarrow e^{i(2x-1)\theta} |x\rangle$$

$$CNOT : |x_1 x_2\rangle \rightarrow |x_1 (x_1 \oplus x_2)\rangle$$

More generally: $|x\rangle \rightarrow \overset{\text{amplitude}}{\phi(x, \rho)} \overset{\text{state}}{|f(x)\rangle}$

Simple Circuit as Polynomial

Circuit

$R_z(\theta) \mathbf{q}_1 ;$

Polynomial

$e^{i(2(0)-1)\theta} |0\rangle$

Simple Circuit as Polynomial

Circuit

$R_z(\theta) \mathbf{q}_1;$

Polynomial

$e^{-i\theta} |0\rangle$

Simple Circuit as Polynomial

Circuit

$R_z(\theta) \quad q_1;$

Polynomial

$$e^{-i\theta} |0\rangle + e^{i\theta} |1\rangle$$

Polynomial Identity Testing (PIT)

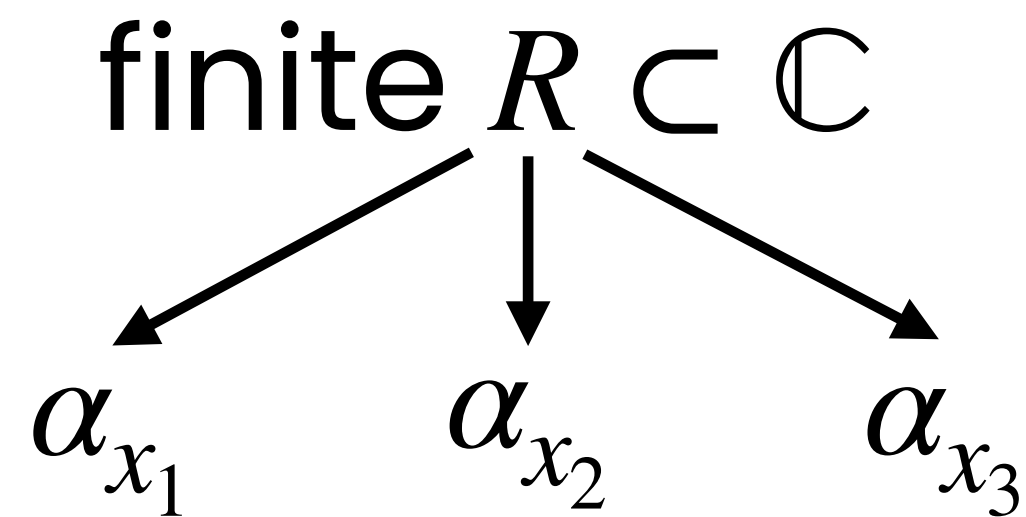
$$\mathbf{x} \in \mathbb{C}^3 \quad p_1(\mathbf{x}) = x_1x_2$$
$$p_2(\mathbf{x}) = x_1x_2 + x_3x_2$$

How can we check $p_1 \equiv p_2$?

Schwartz-Zippel Lemma

$$p_1(\mathbf{x}) = x_1x_2$$

$$p_2(\mathbf{x}) = x_1x_2 + x_3x_2$$



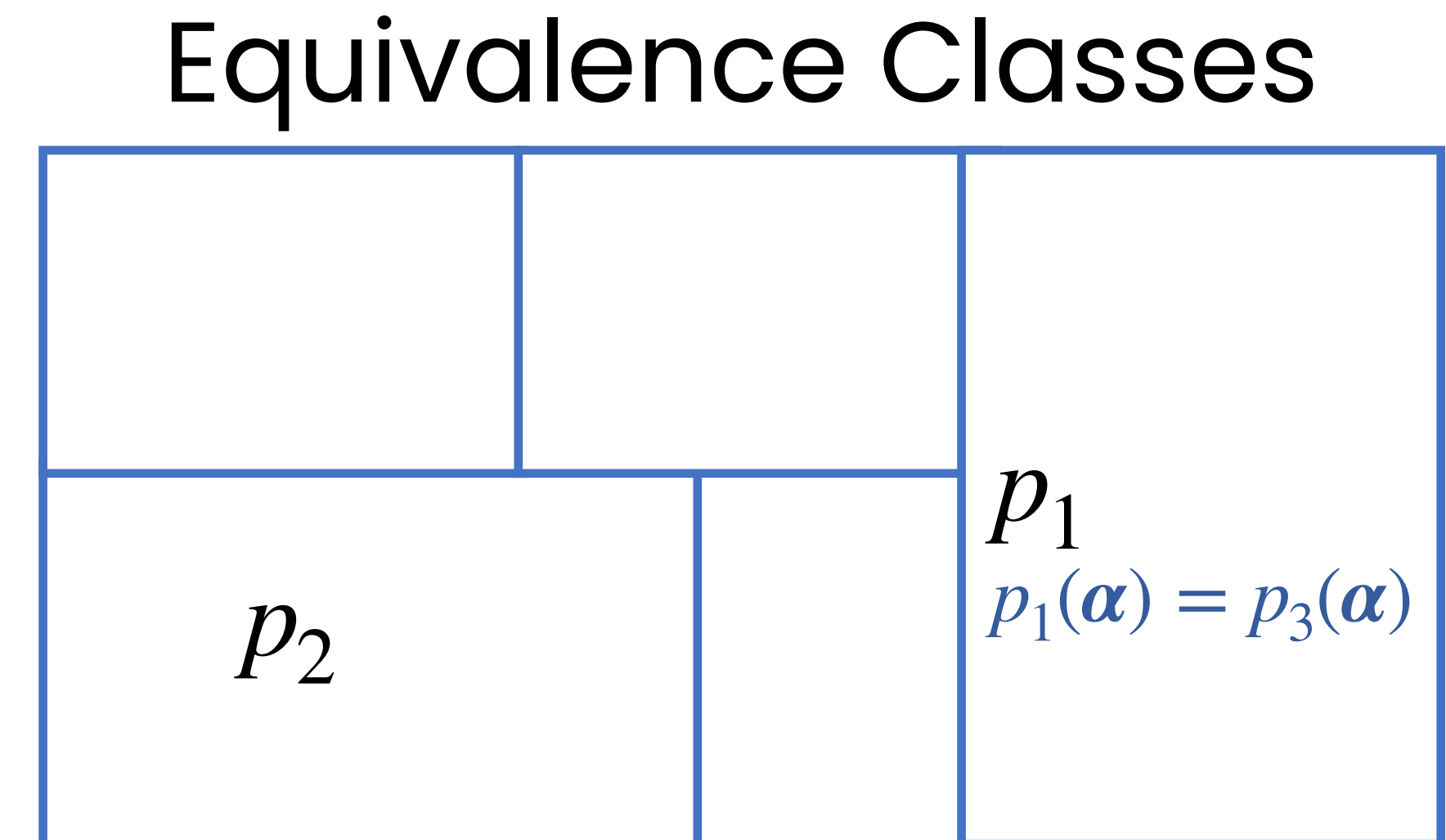
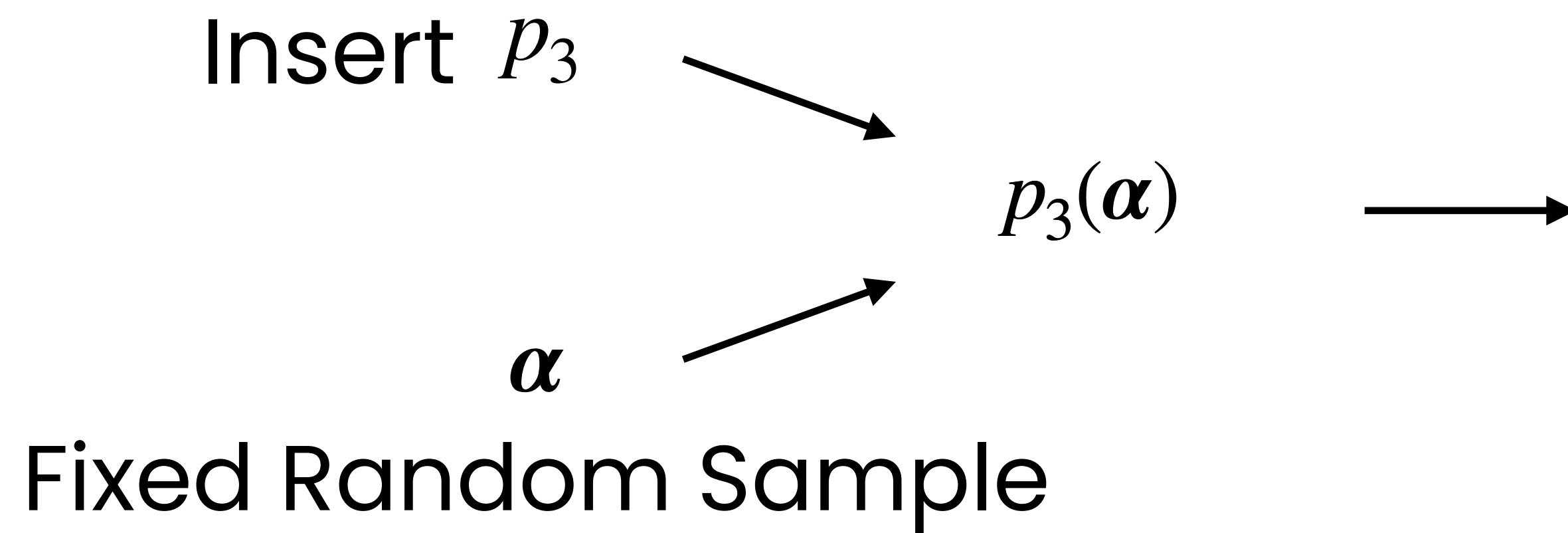
$$p_1(\alpha) = \alpha_{x_1}\alpha_{x_2}$$

$$p_2(\alpha) = \alpha_{x_1}\alpha_{x_2} + \alpha_{x_3}\alpha_{x_2}$$

Check $p_1(\alpha) = p_2(\alpha)$

If $p_1 \neq p_2$, then the probability that $p_1(\alpha) = p_2(\alpha)$ is at most $\frac{\text{small}}{\text{very large}}$ $\frac{d \text{ (max total degree)}}{|R|}$

Polynomial Identity Filter (PIF)



QUESO's Improvements

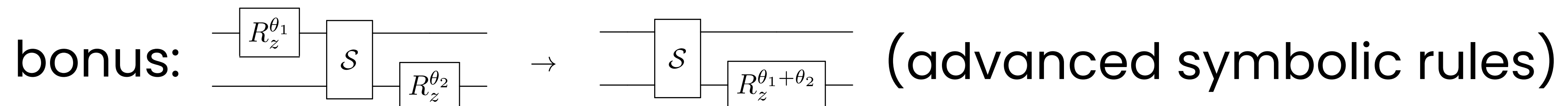
```
rules = []
circuits = enumerate(max_qubits, max_size)
for c1 in circuits:
    for c2 in circuits:
        if verify(c1, c2):
            insert into PIF
            rules.append(c1 → c2)
```

big (pointing to `circuits`)

uh oh... (pointing to the nested loop structure)

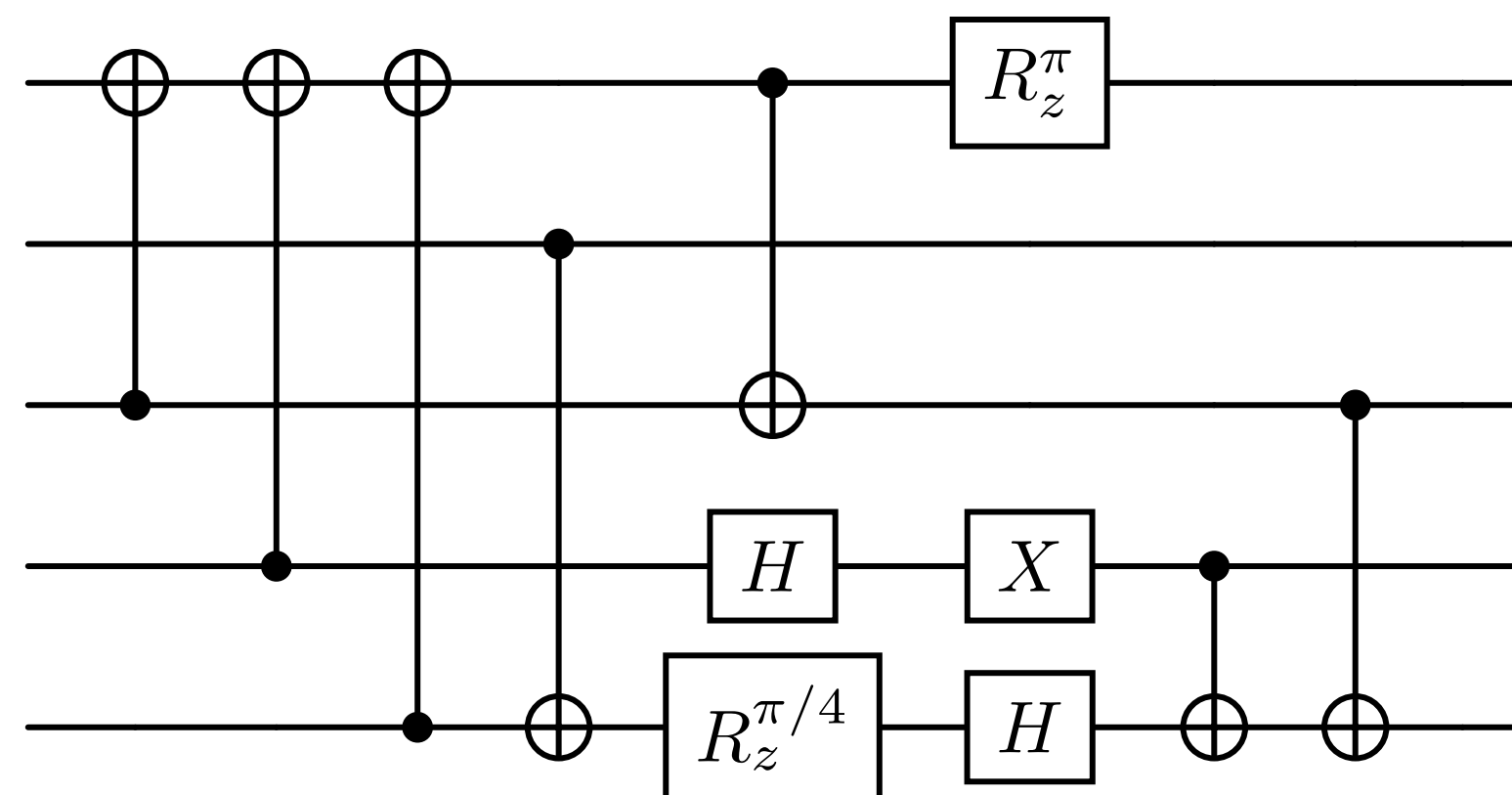
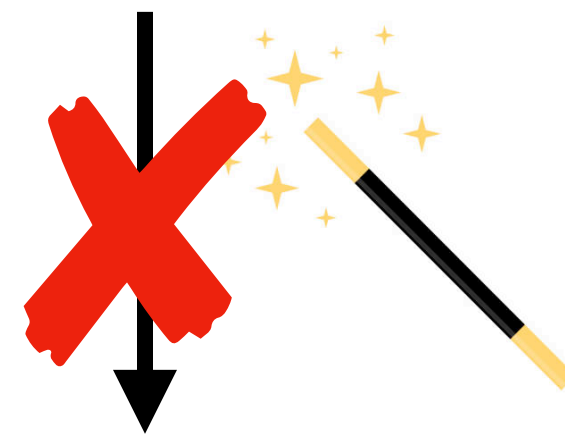
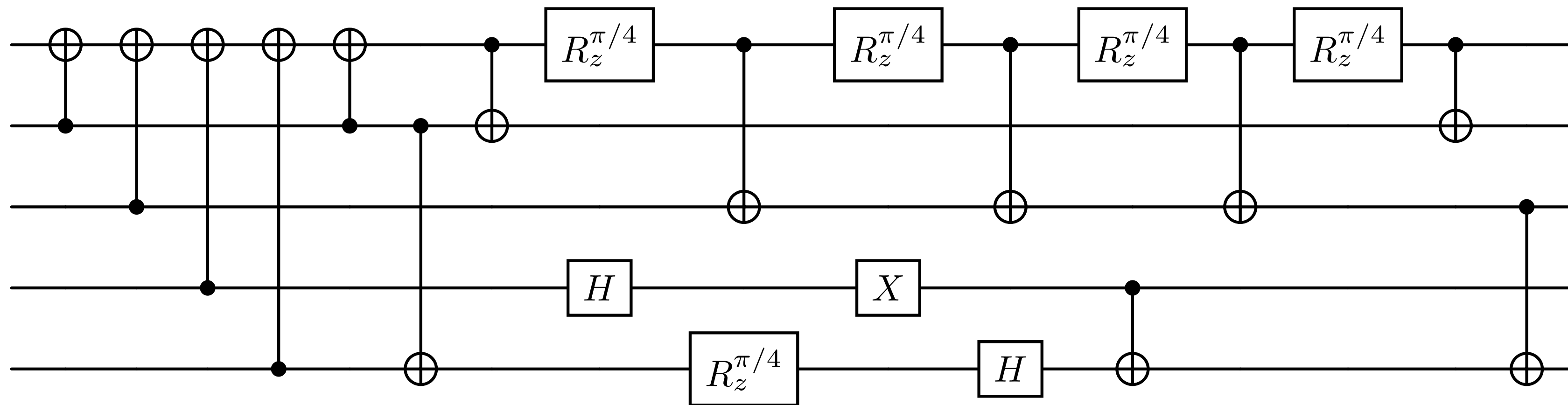
expensive (underlined, pointing to `verify(c1, c2)`)

Efficient equivalence check between circuit pairs!

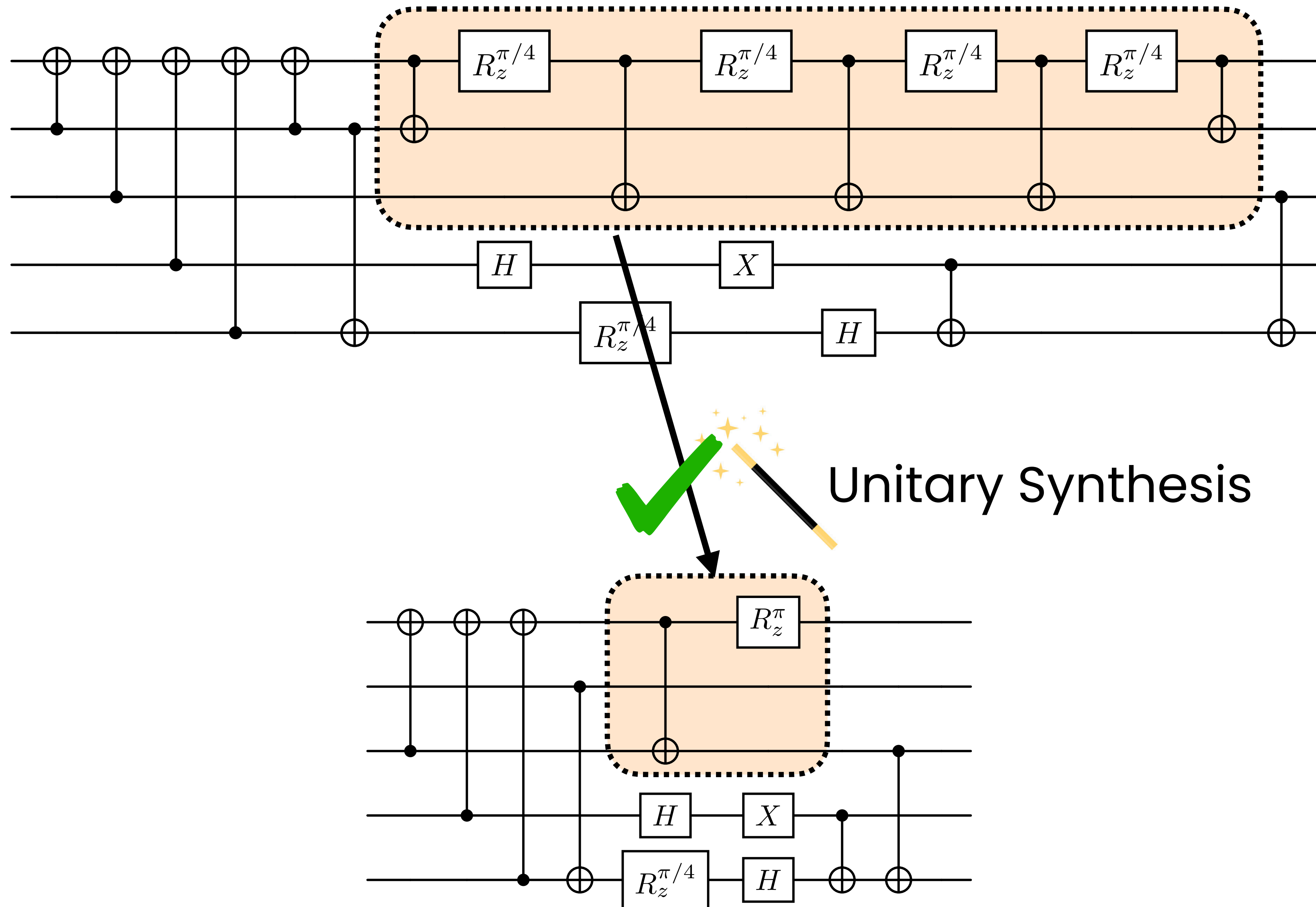


Circuit Resynthesis

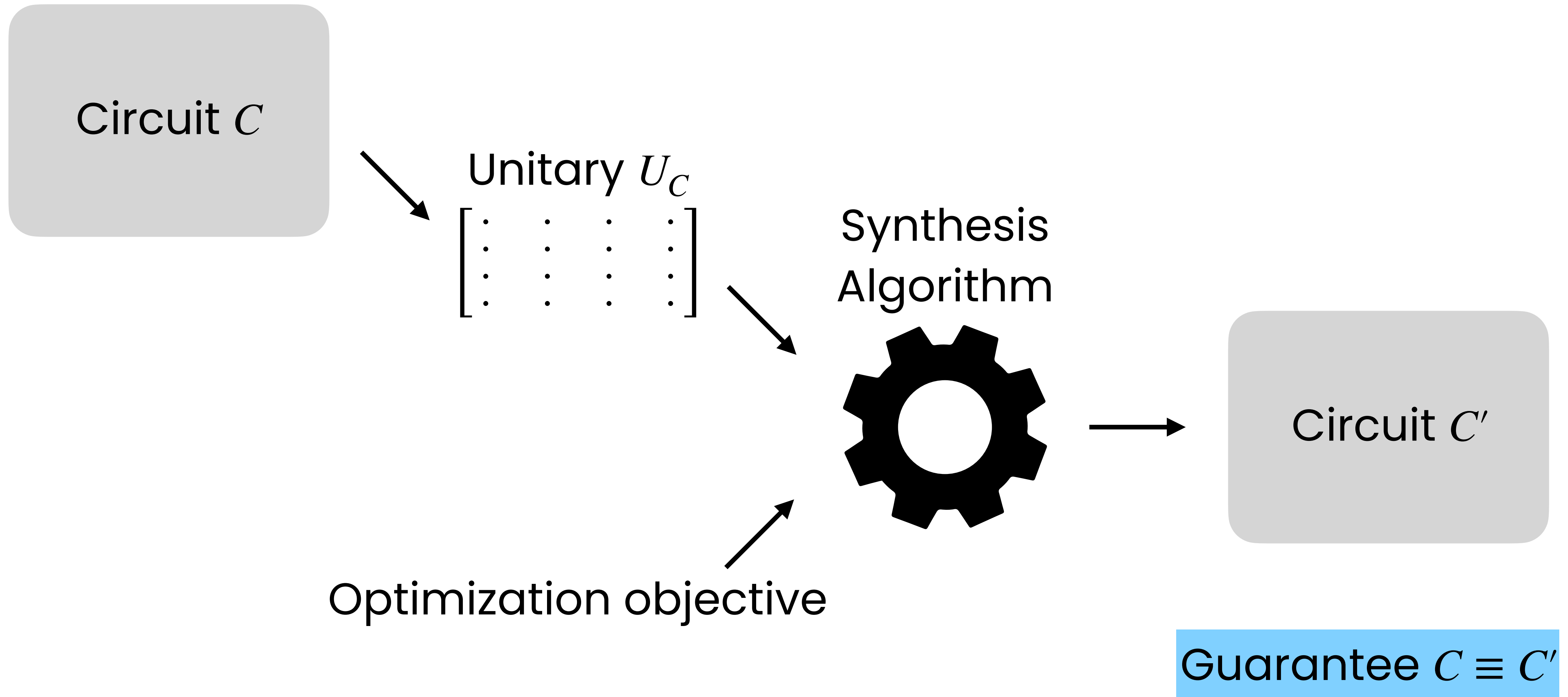
Circuit Resynthesis



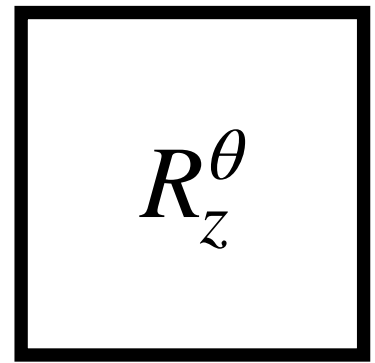
Circuit Resynthesis



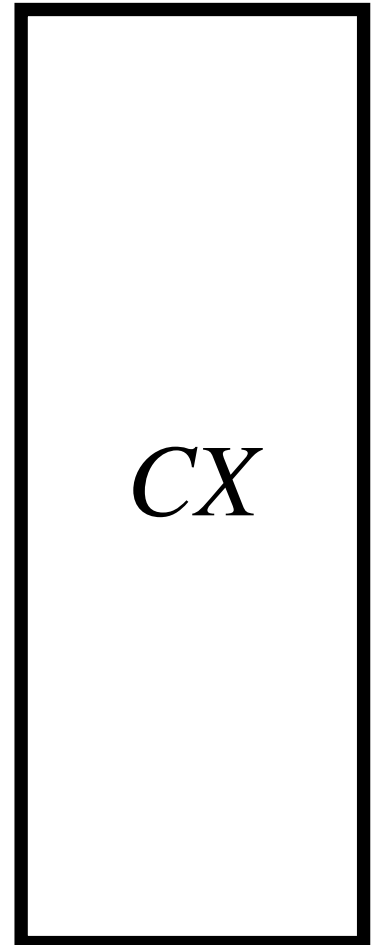
Unitary Synthesis



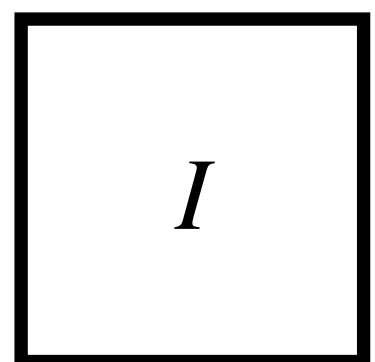
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

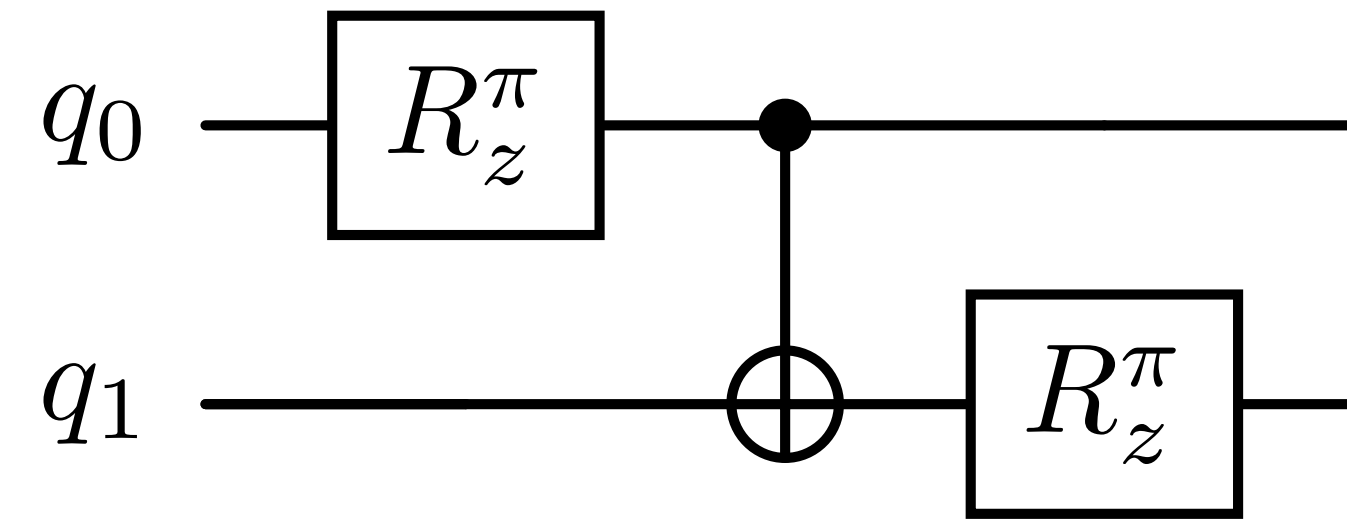


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

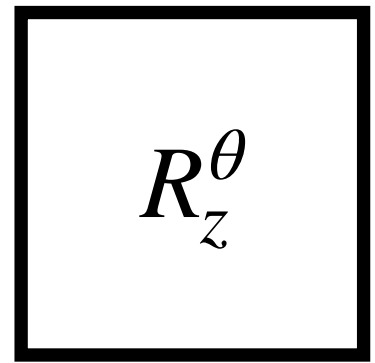


$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

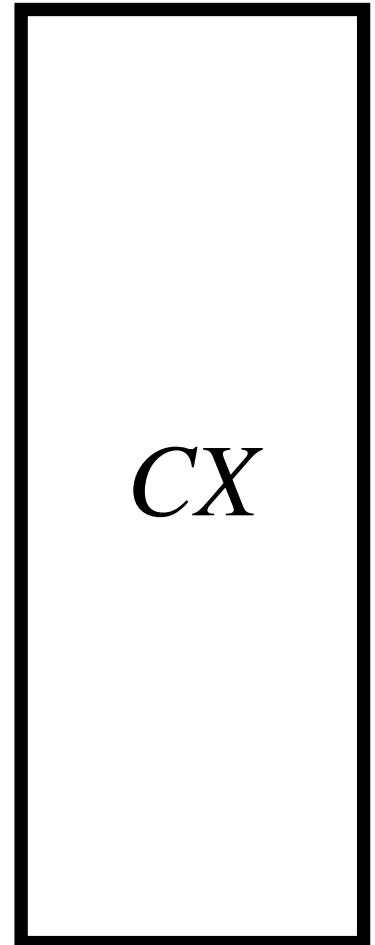
$C :=$



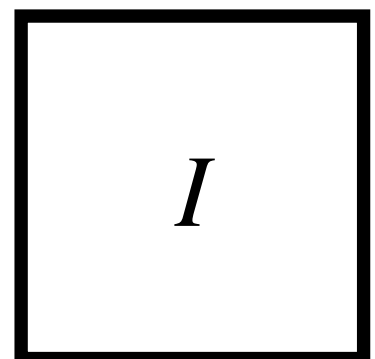
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

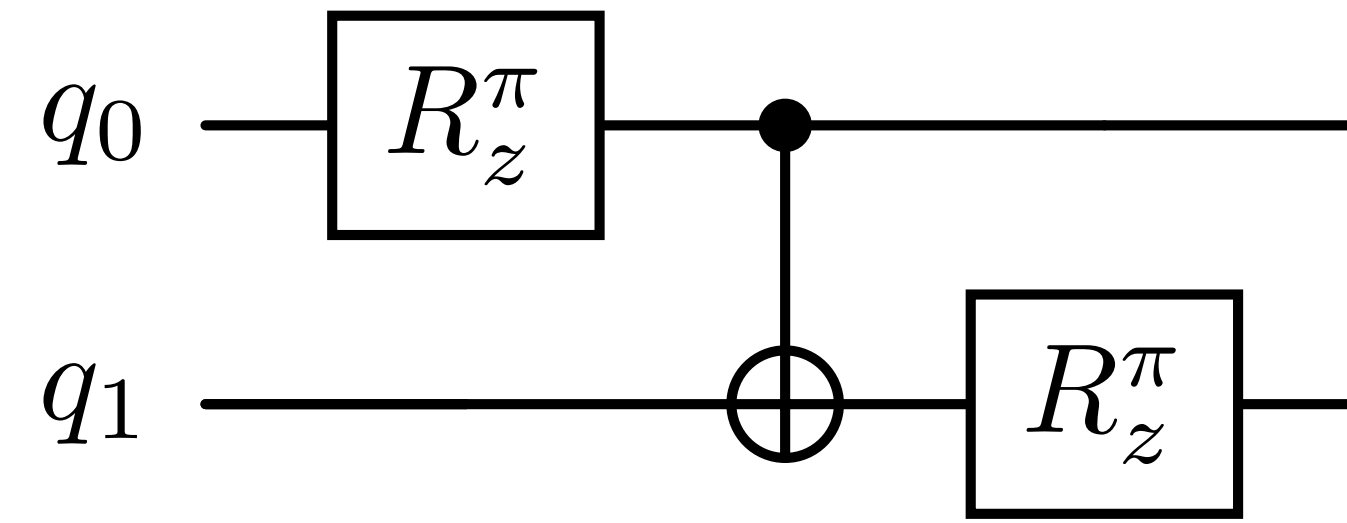


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

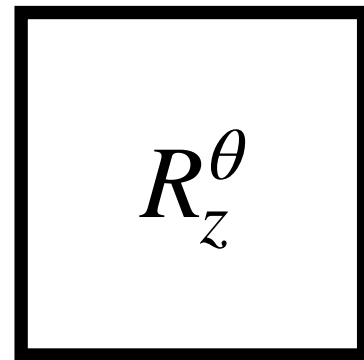
$C :=$



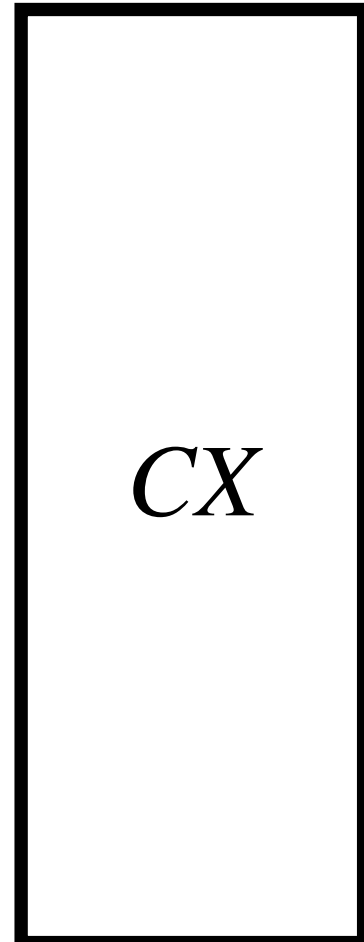
$U_C =$

$U_{R_z^\pi}$

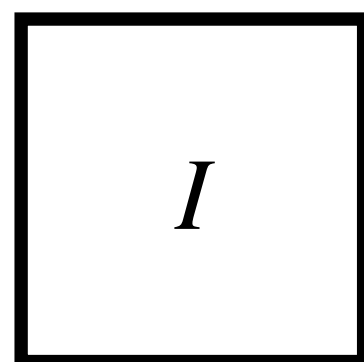
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

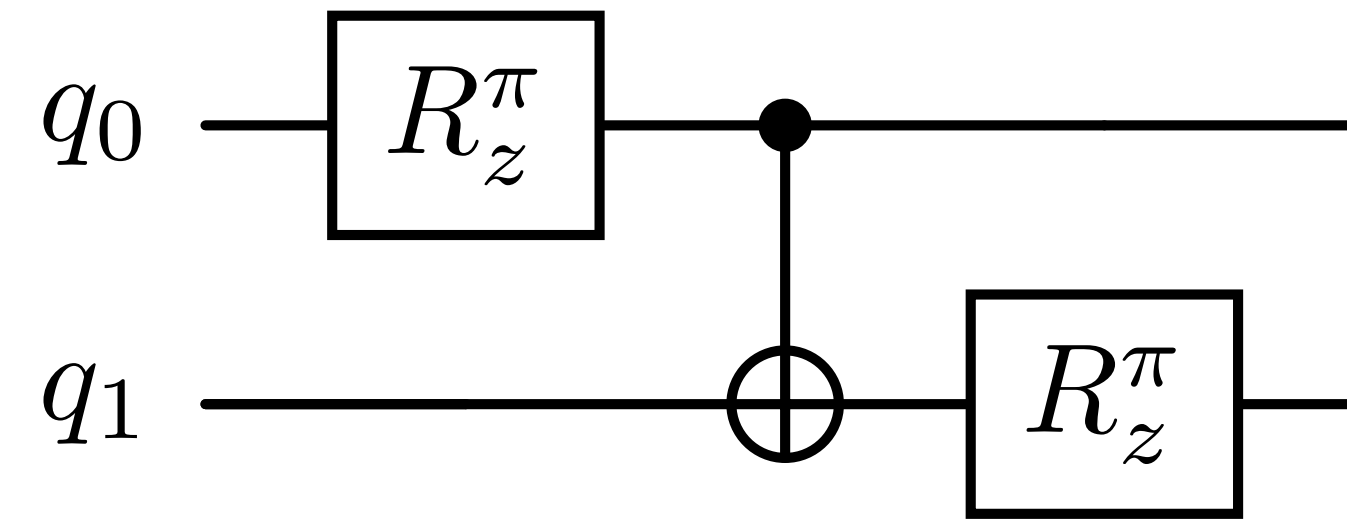


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

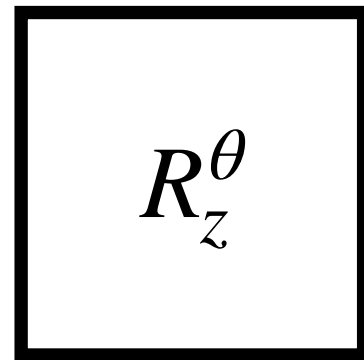
$$C :=$$



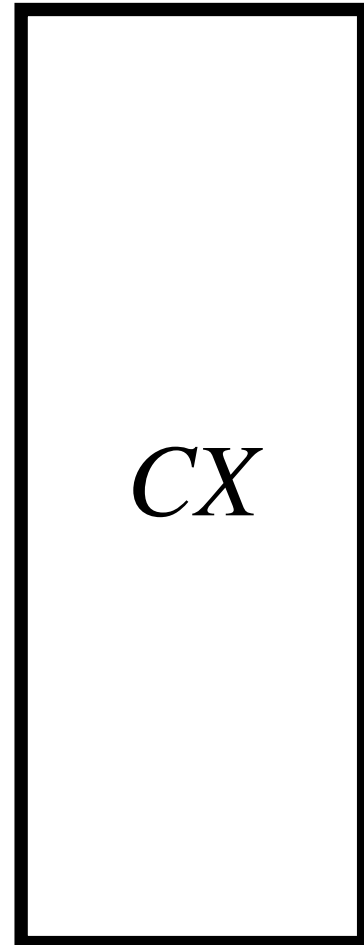
$$U_C = 2^2 \times 2^2$$

$$U_{R_z^\pi}$$

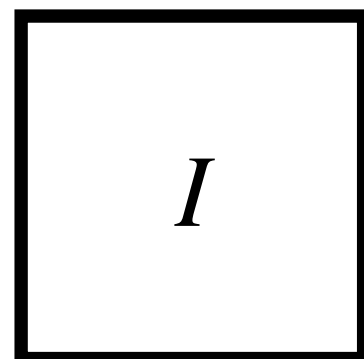
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

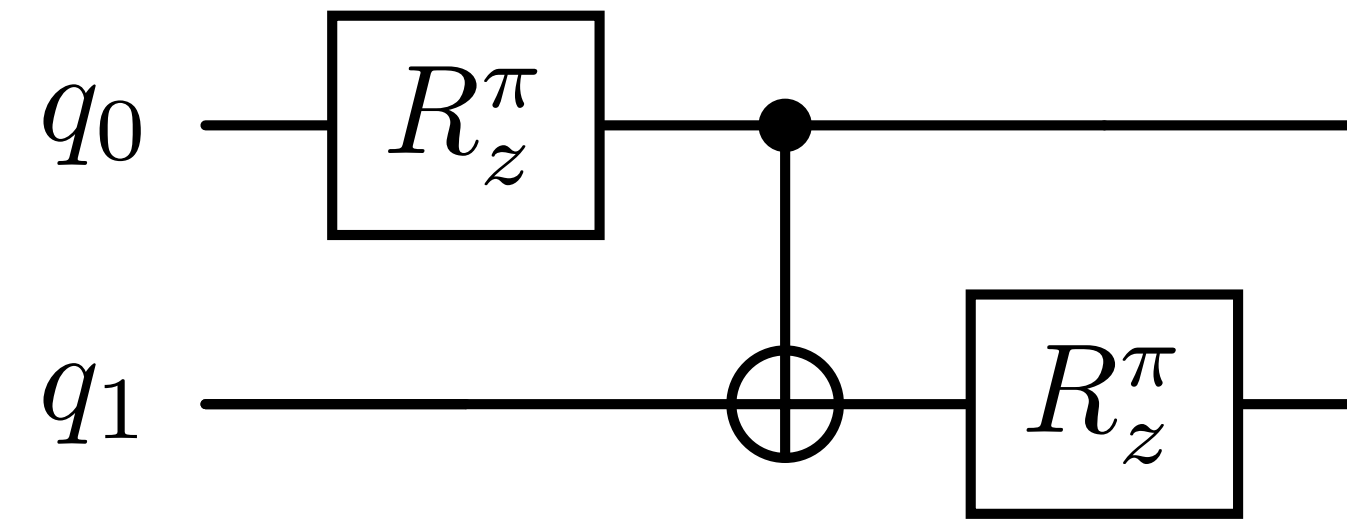


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

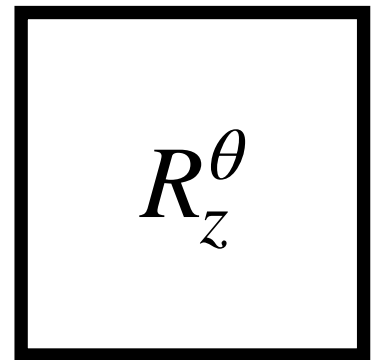
$$C :=$$



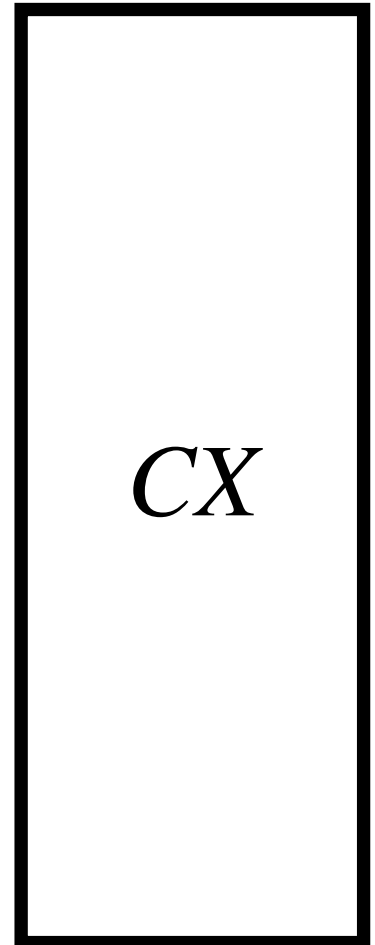
$$U_C = 2^2 \times 2^2$$

$$U_{R_z^\pi} = 2 \times 2$$

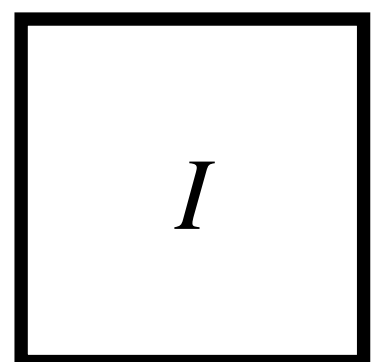
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

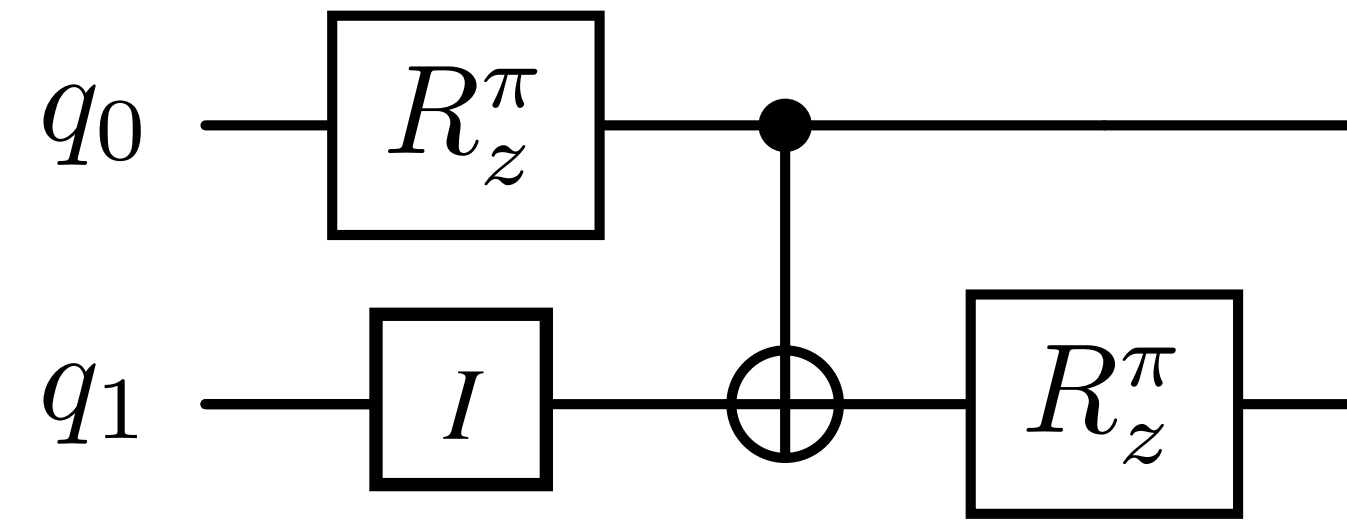


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

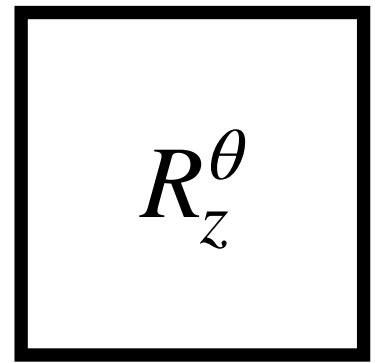
$$C :=$$



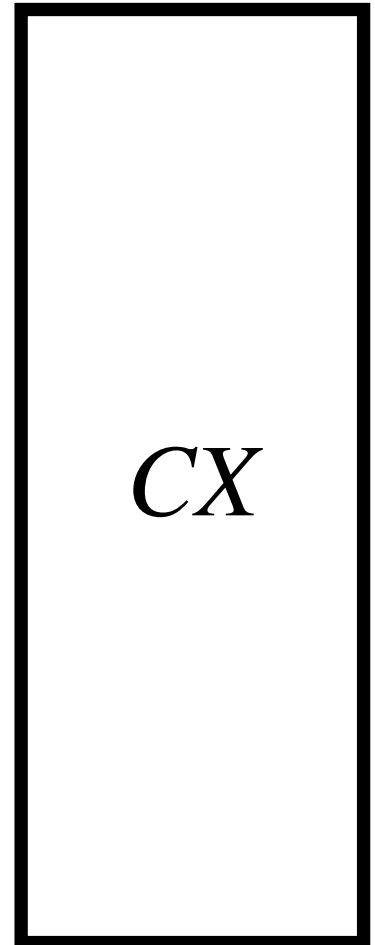
$$U_C = 2^2 \times 2^2$$

$$U_{R_z^\pi} = 2 \times 2$$

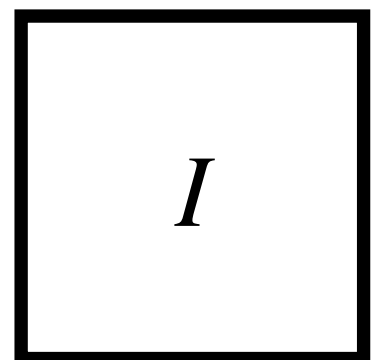
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

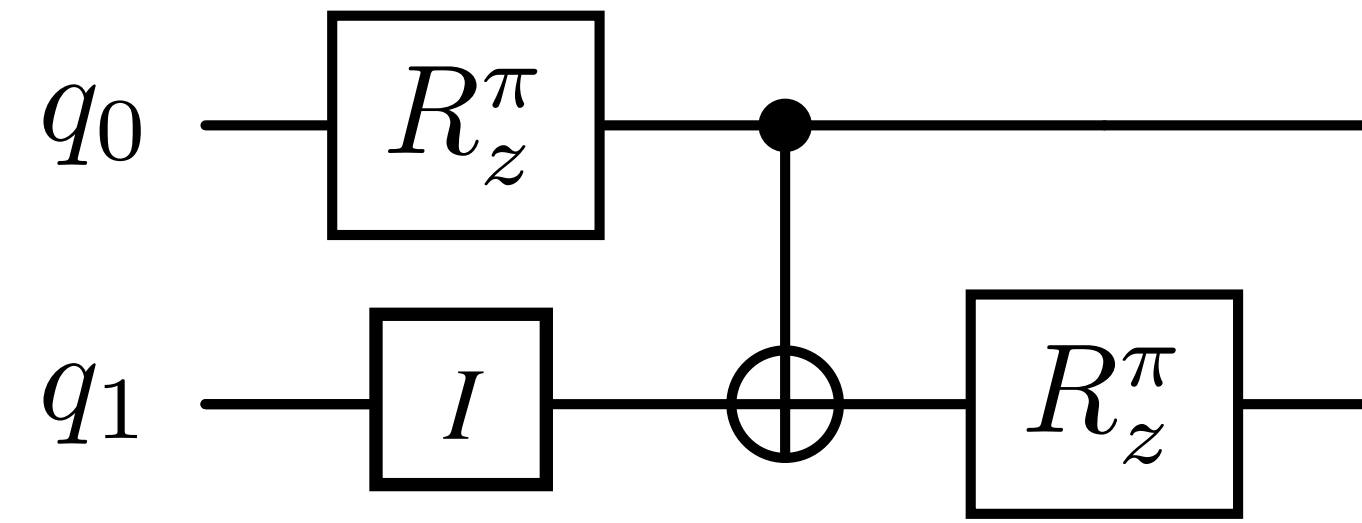


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C :=$$



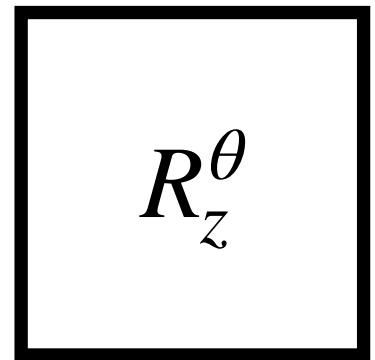
$$U_C =$$

$$2^2 \times 2^2$$

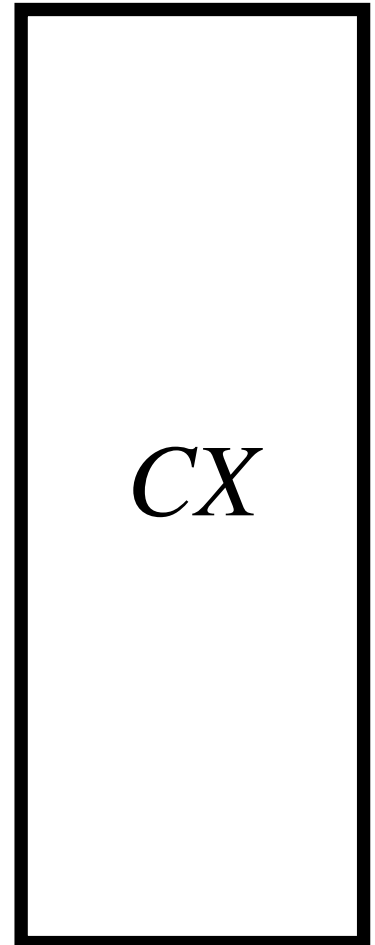
$$(U_{R_z^\pi} \otimes U_I)$$

$$2 \times 2$$

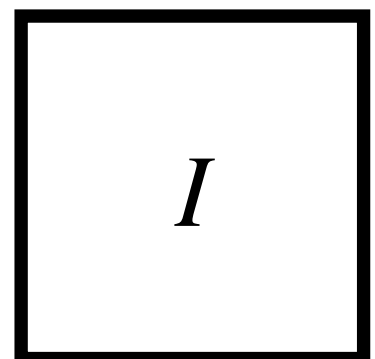
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

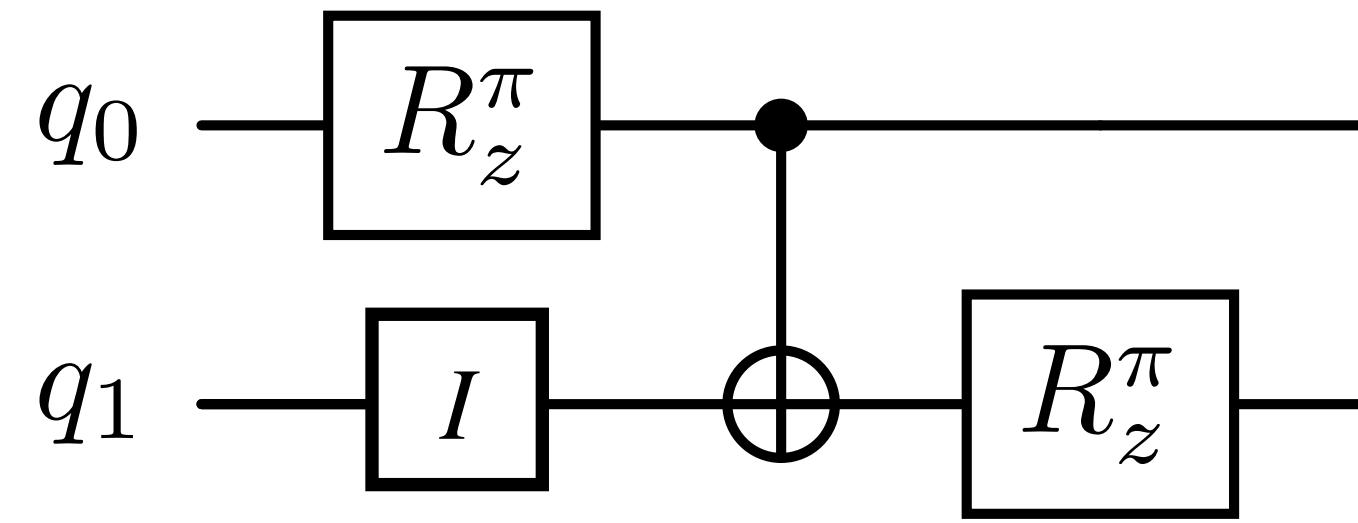


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C :=$$



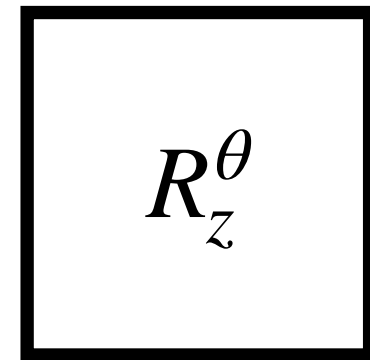
$$U_C =$$

$$2^2 \times 2^2$$

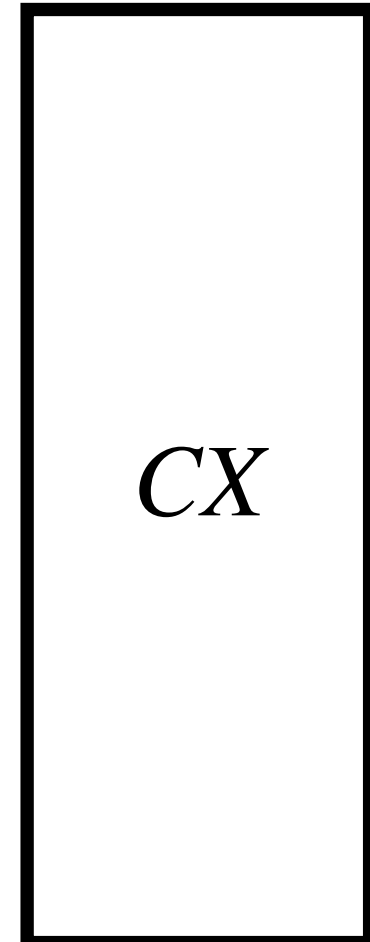
$$(U_{R_z^\pi} \otimes U_I)$$

$$\frac{2 \times 2 \quad 2 \times 2}{2^2 \times 2^2}$$

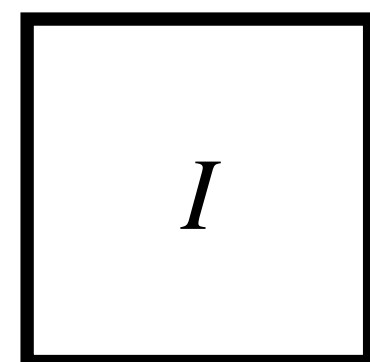
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

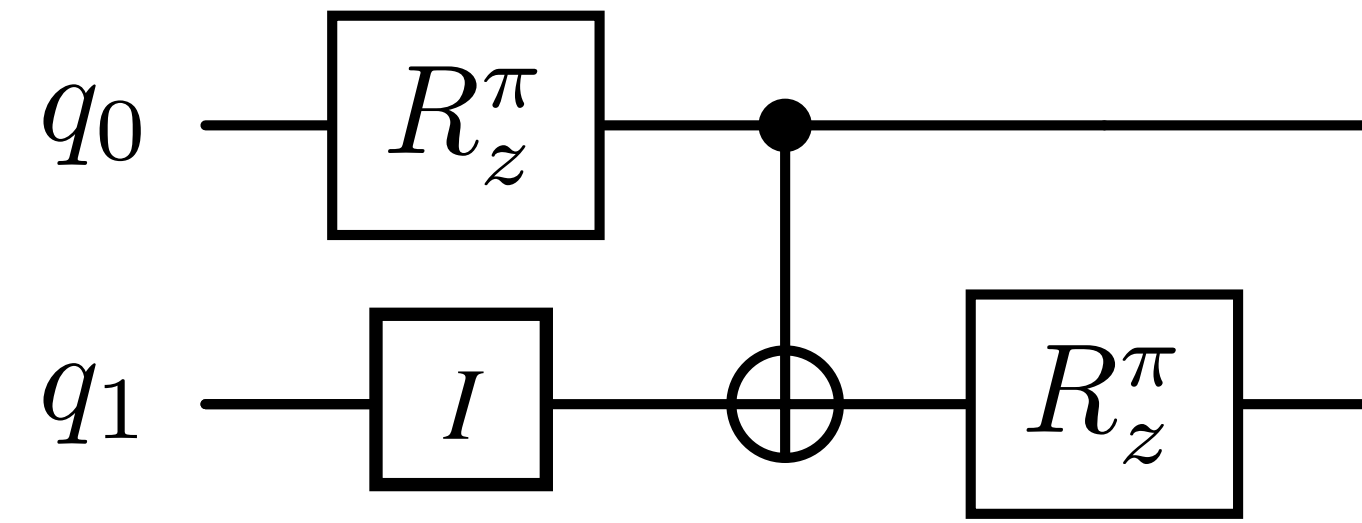


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C :=$$



$$U_C =$$

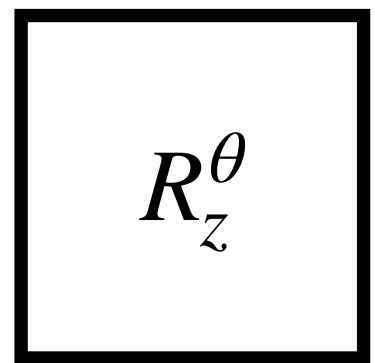
$$2^2 \times 2^2$$

$$(U_{R_z^\pi} \otimes U_I)$$

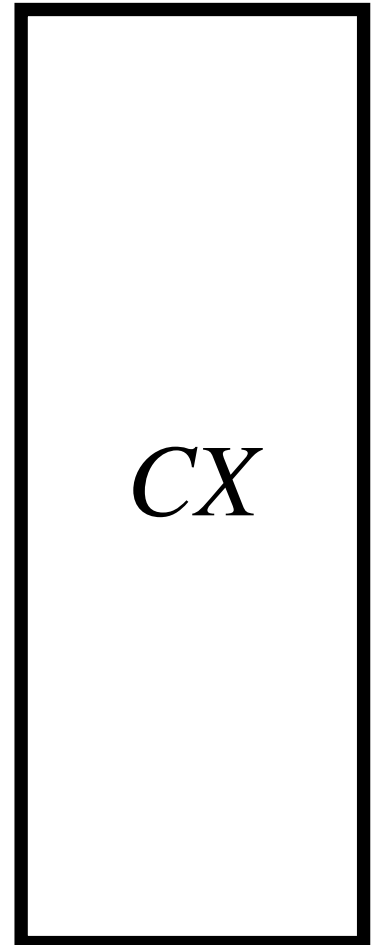
$$\frac{2 \times 2 \quad 2 \times 2}{2^2 \times 2^2}$$

$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} e^{-i\frac{\pi}{2}} \cdot U_I & 0 \cdot U_I \\ 0 \cdot U_I & e^{i\frac{\pi}{2}} \cdot U_I \end{bmatrix}$$

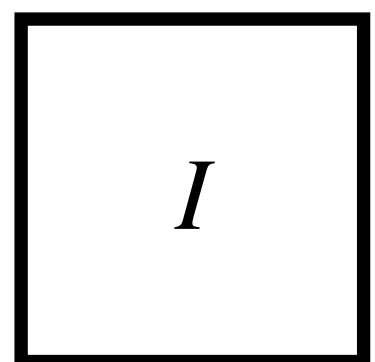
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

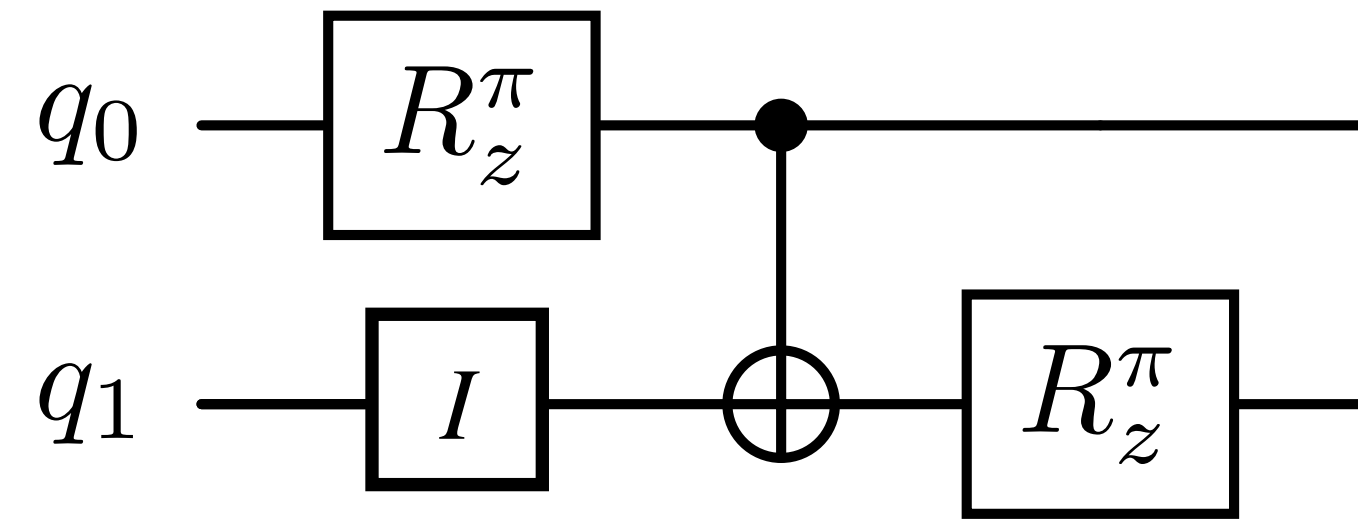


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C :=$$



$$U_C =$$

$2^2 \times 2^2$

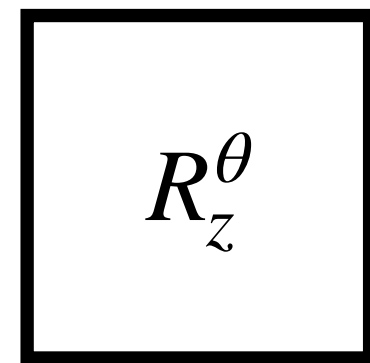
$$(U_{R_z^\pi} \otimes U_I)$$

$\frac{2 \times 2 \quad 2 \times 2}{2^2 \times 2^2}$

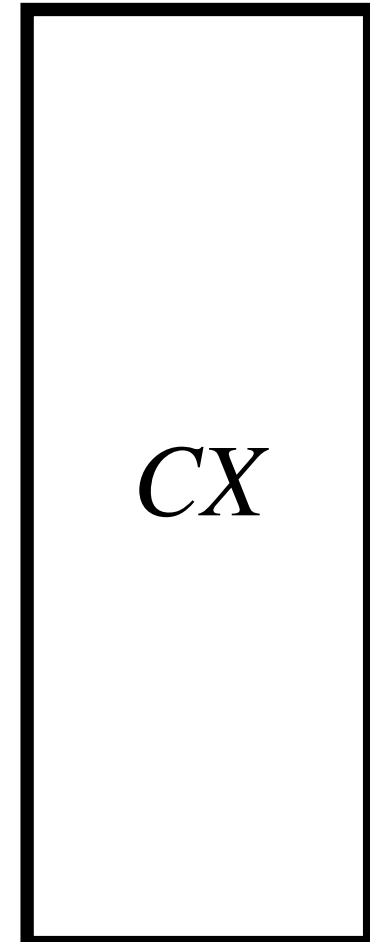
$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} e^{-i\frac{\pi}{2}} \cdot U_I & 0 \cdot U_I \\ 0 \cdot U_I & e^{i\frac{\pi}{2}} \cdot U_I \end{bmatrix}$$

$2 \times 2 \quad 2 \times 2$

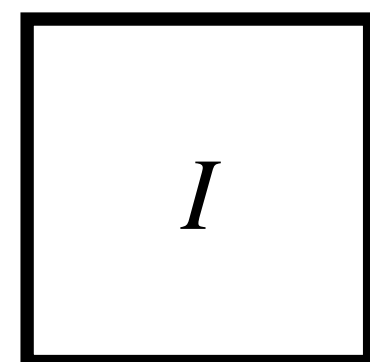
Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

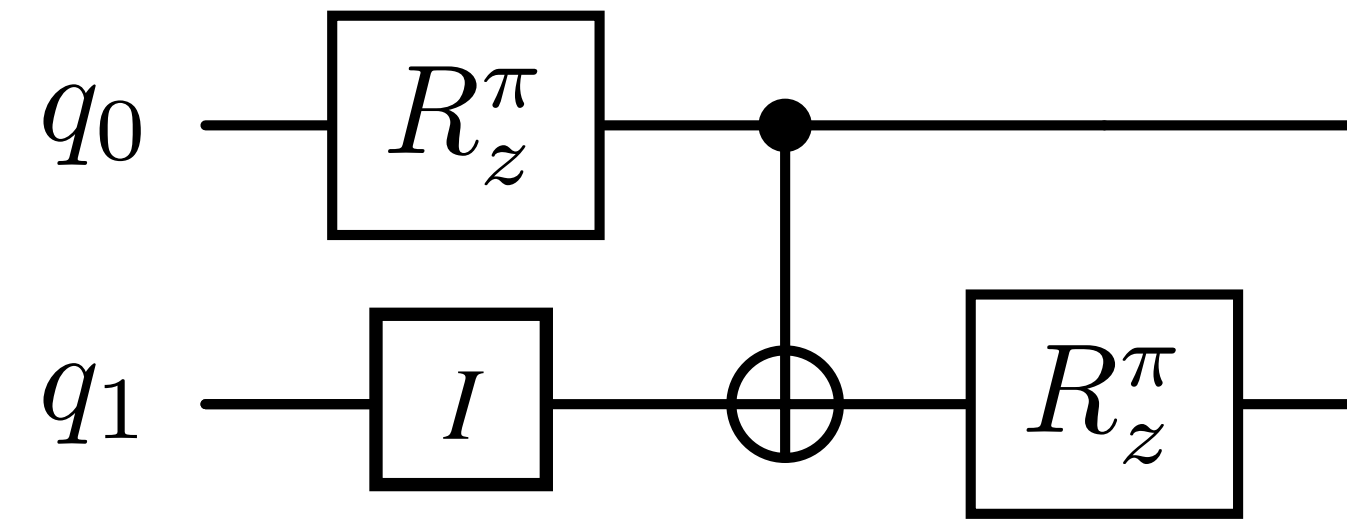


$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C :=$$



$$U_C =$$

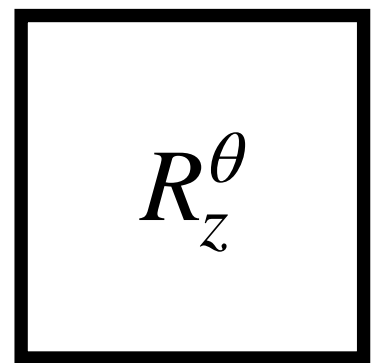
$2^2 \times 2^2$

$$U_{CX} \cdot \underbrace{(U_{R_z^\pi} \otimes U_I)}_{2^2 \times 2^2}$$

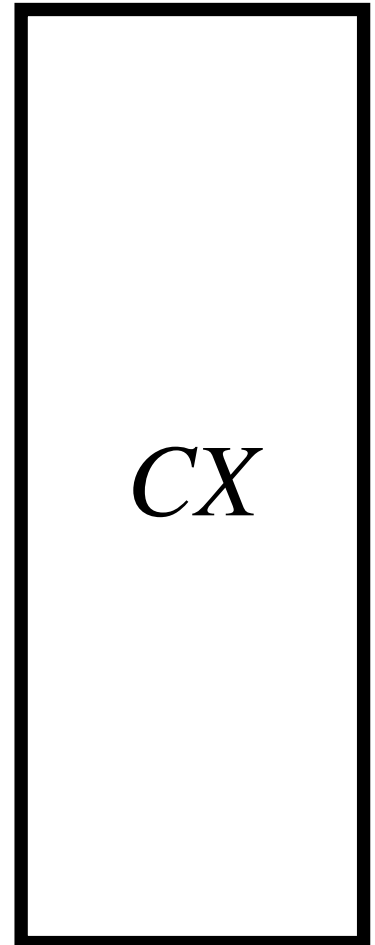
$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} e^{-i\frac{\pi}{2}} \cdot U_I & 0 \cdot U_I \\ 0 \cdot U_I & e^{i\frac{\pi}{2}} \cdot U_I \end{bmatrix}$$

$2 \times 2 \quad 2 \times 2$

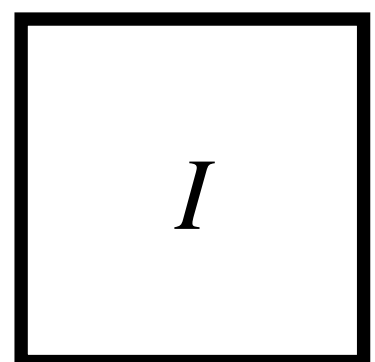
Circuit to Unitary



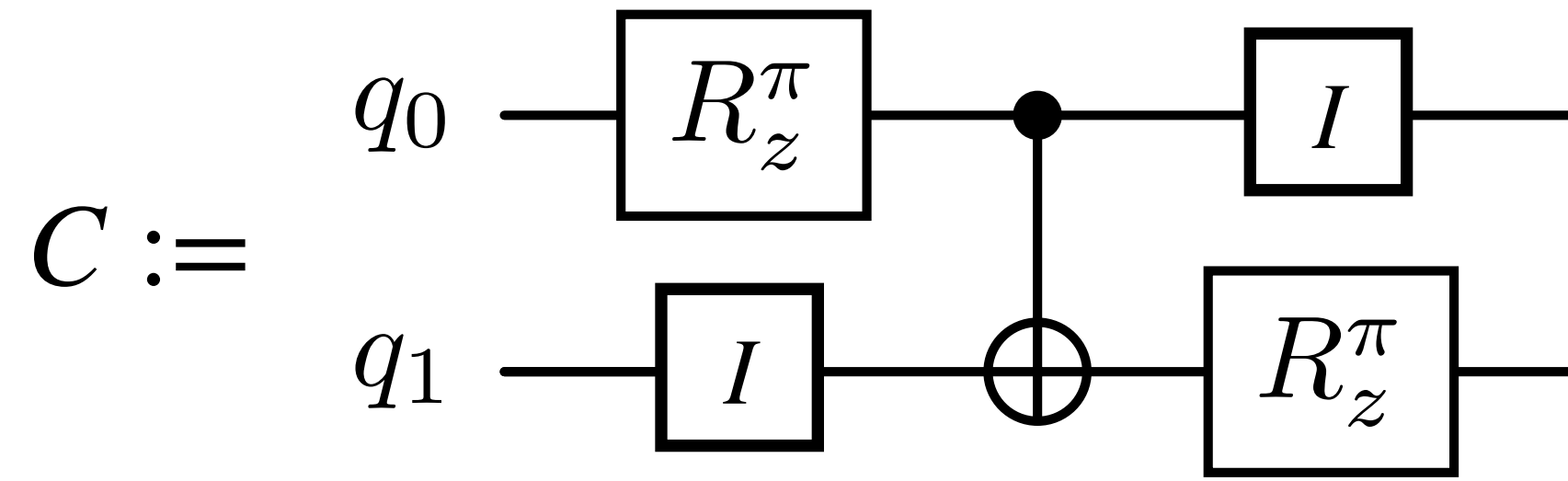
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



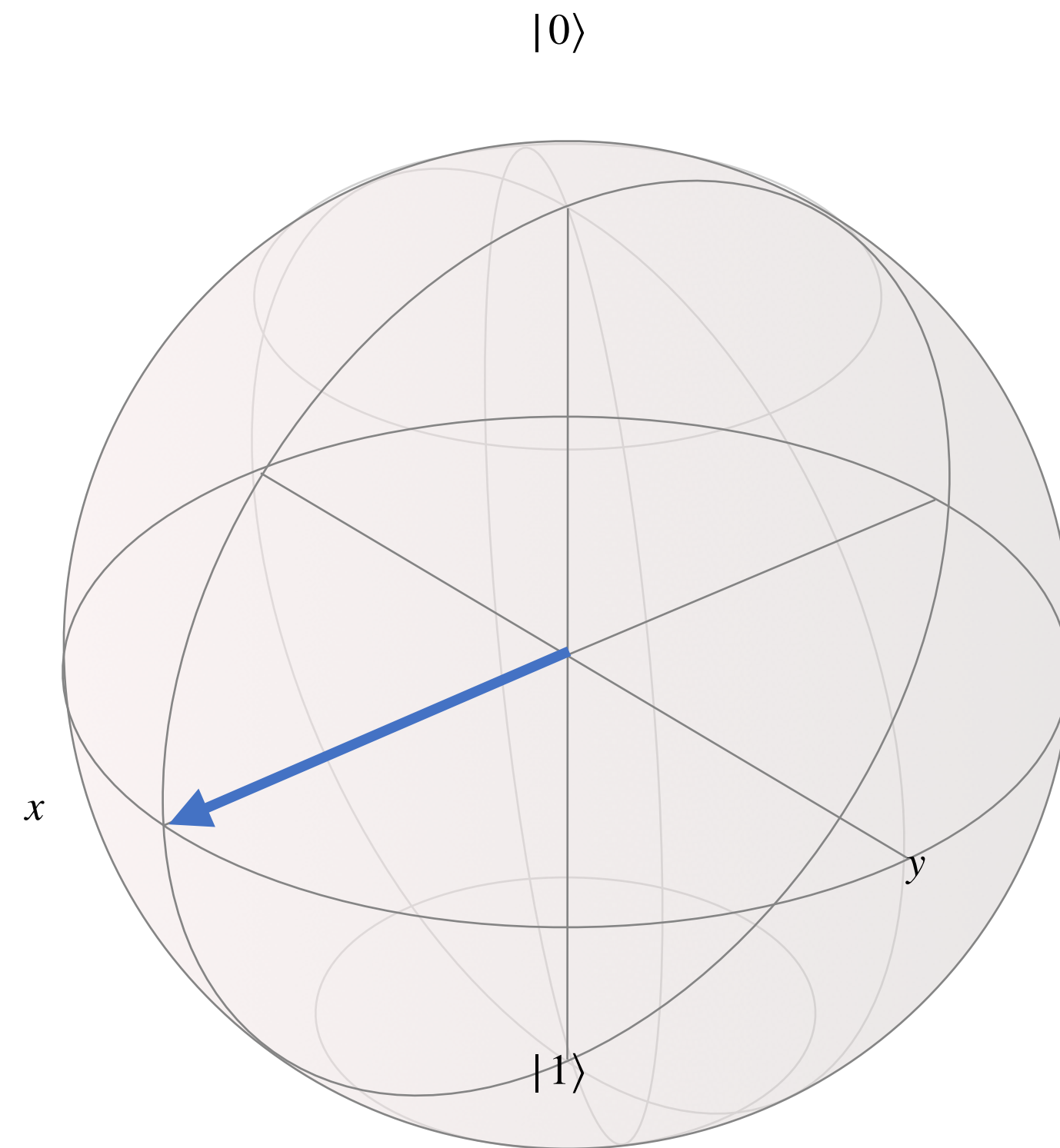
$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



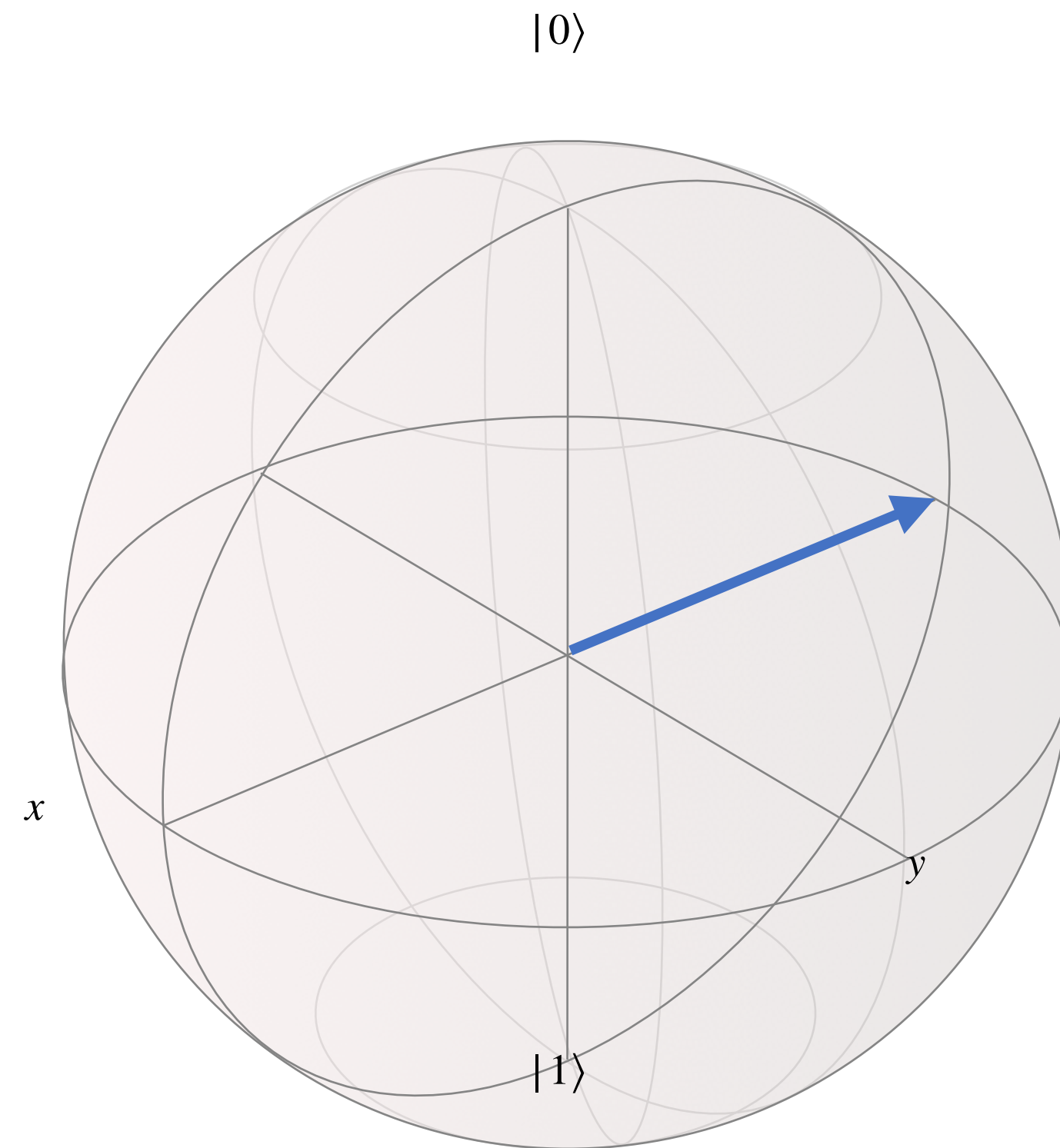
$$U_C = (U_I \otimes U_{R_z^\pi}) \cdot U_{CX} \cdot \underbrace{(U_{R_z^\pi} \otimes U_I)}_{2^2 \times 2^2}$$

$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} e^{-i\frac{\pi}{2}} \cdot U_I & 0 \cdot U_I \\ 0 \cdot U_I & e^{i\frac{\pi}{2}} \cdot U_I \end{bmatrix}$$

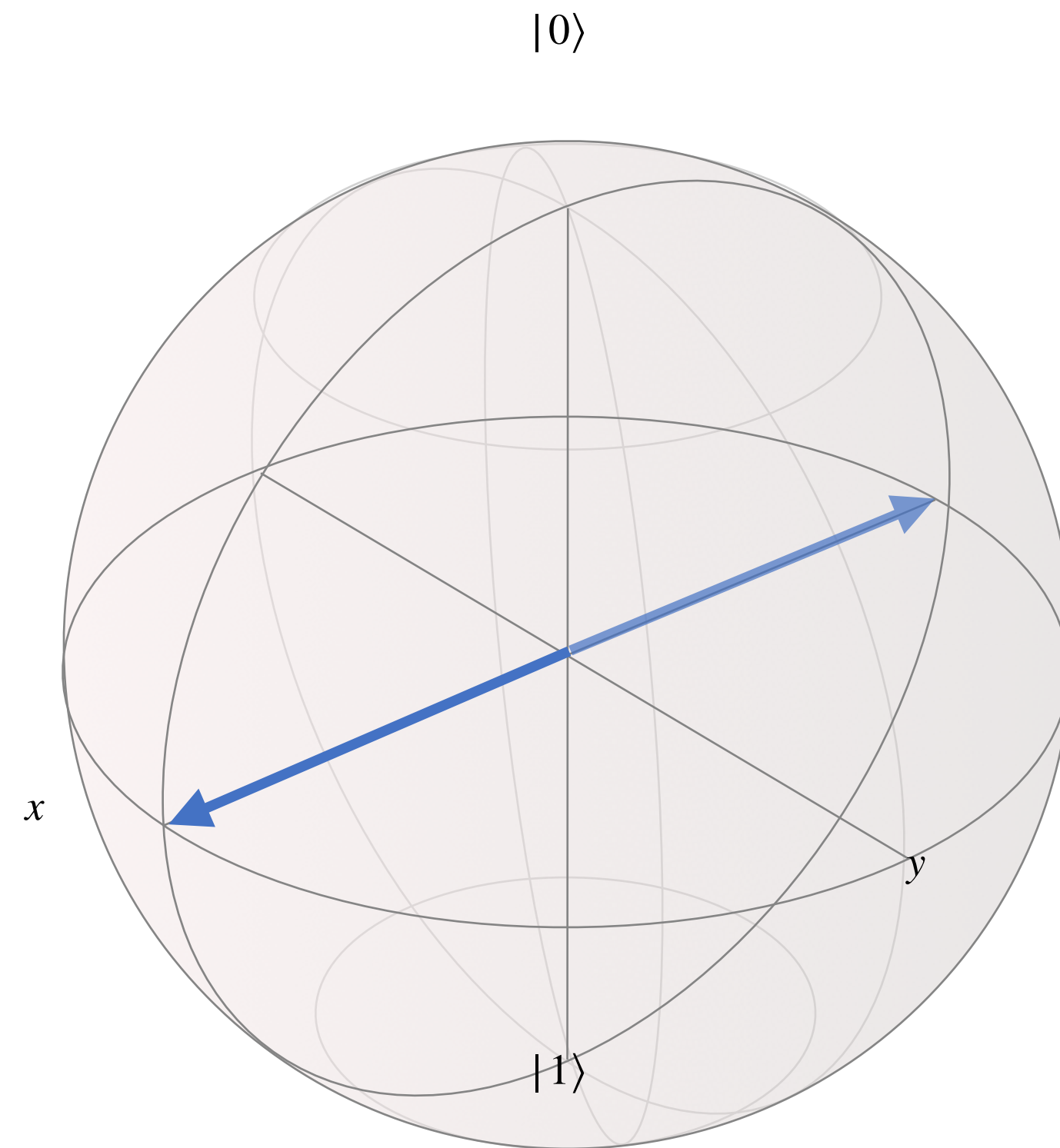
Approximate Circuit Equivalence



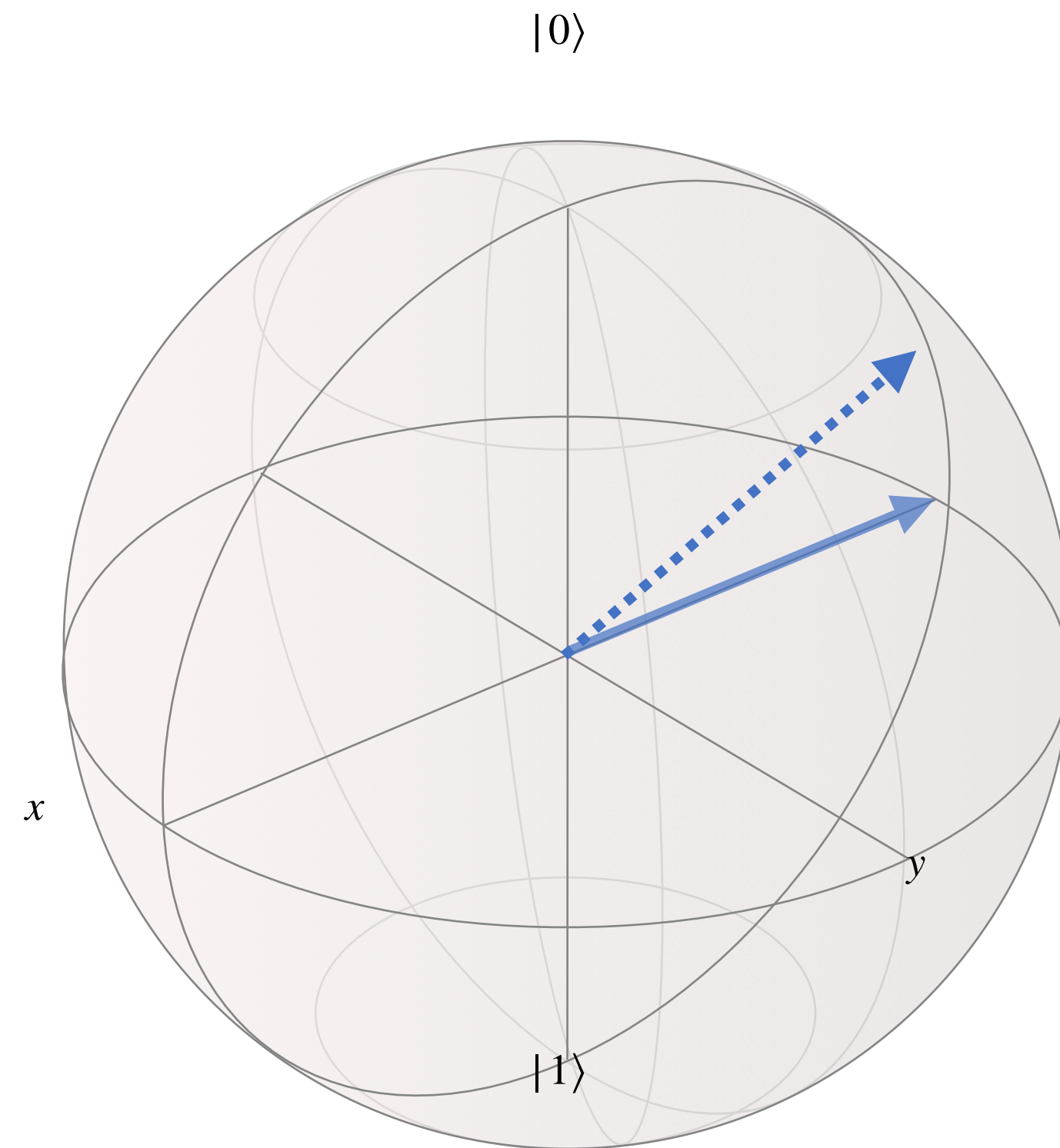
Approximate Circuit Equivalence



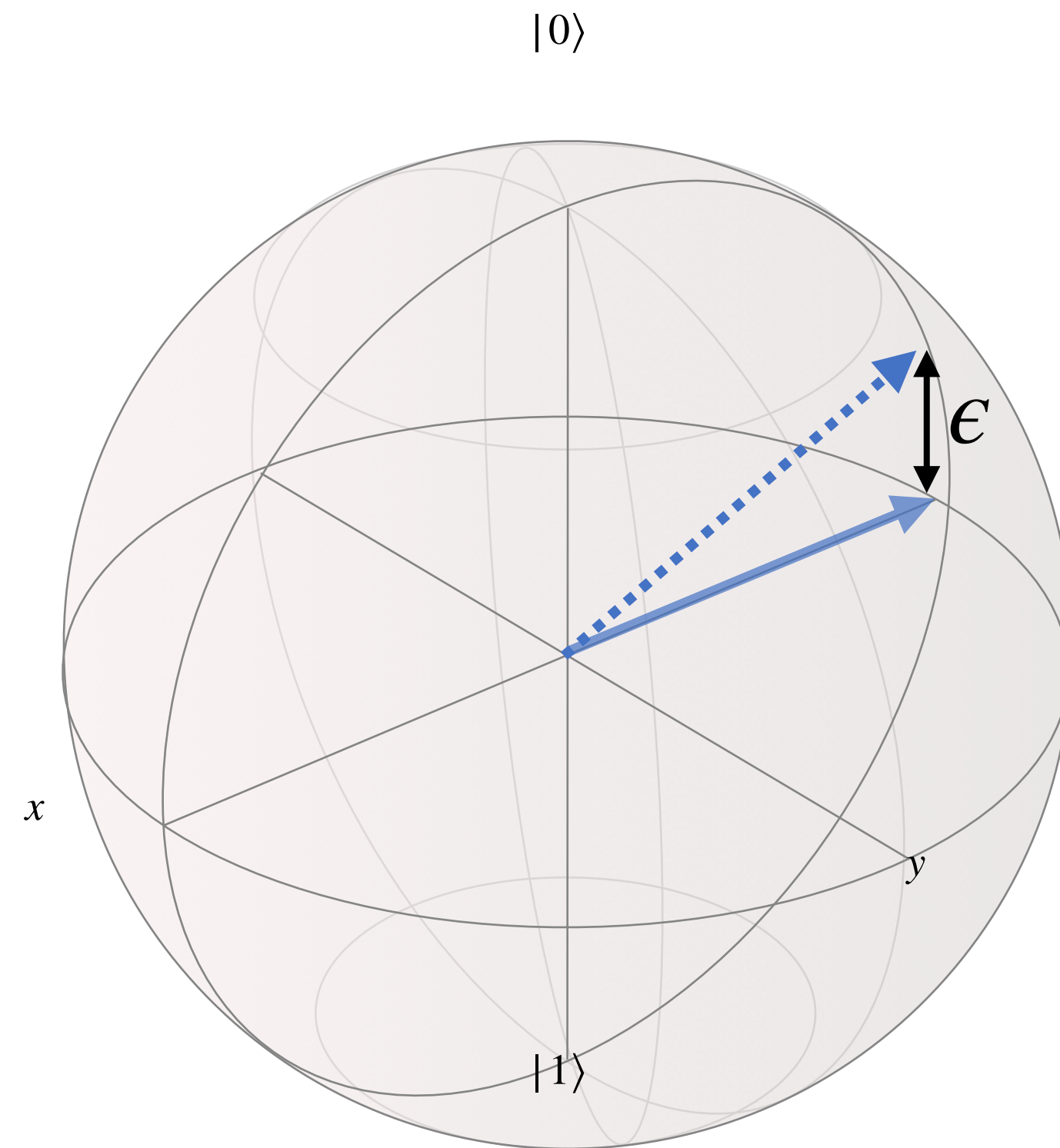
Approximate Circuit Equivalence



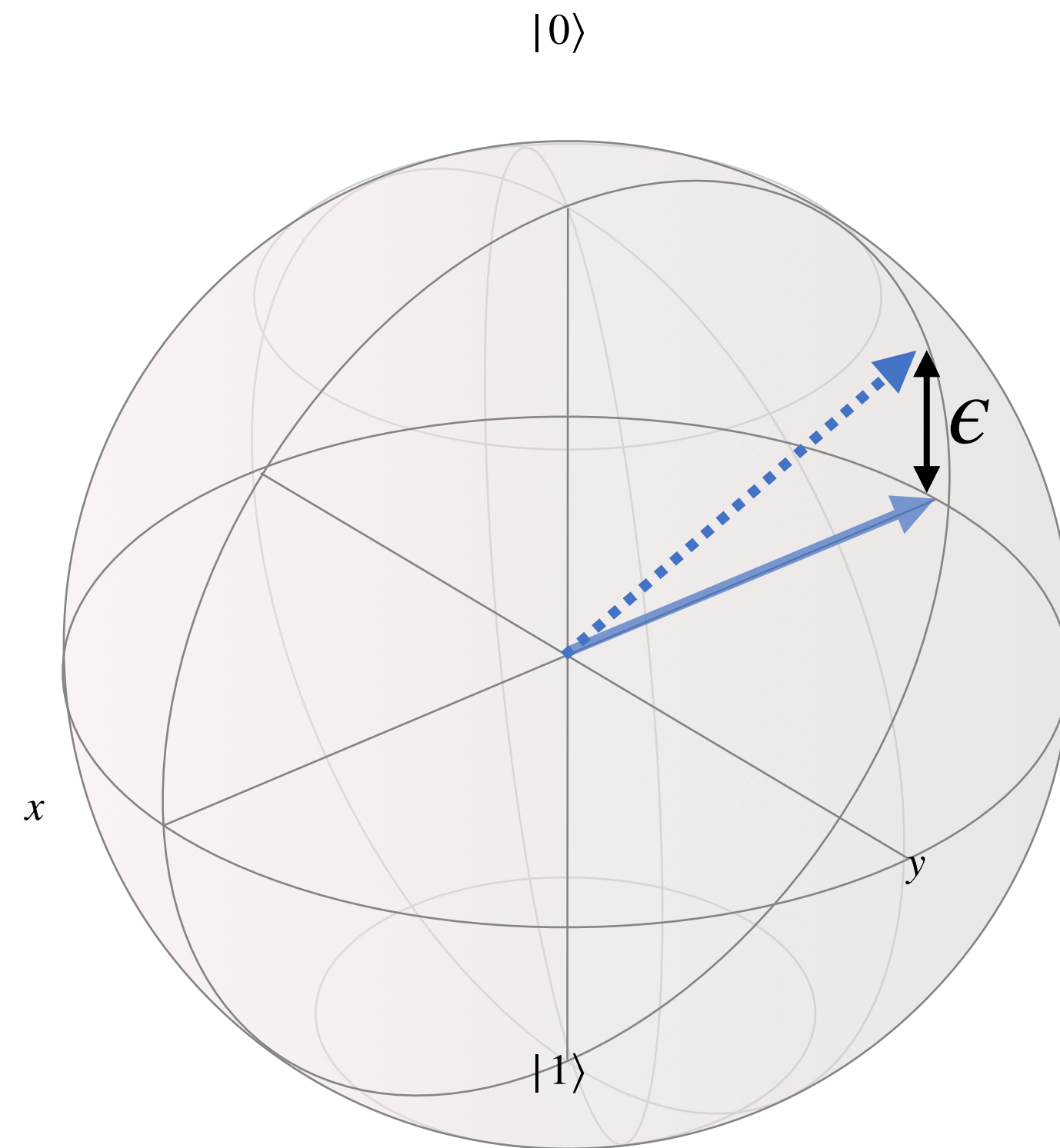
Approximate Circuit Equivalence



Approximate Circuit Equivalence



Approximate Circuit Equivalence



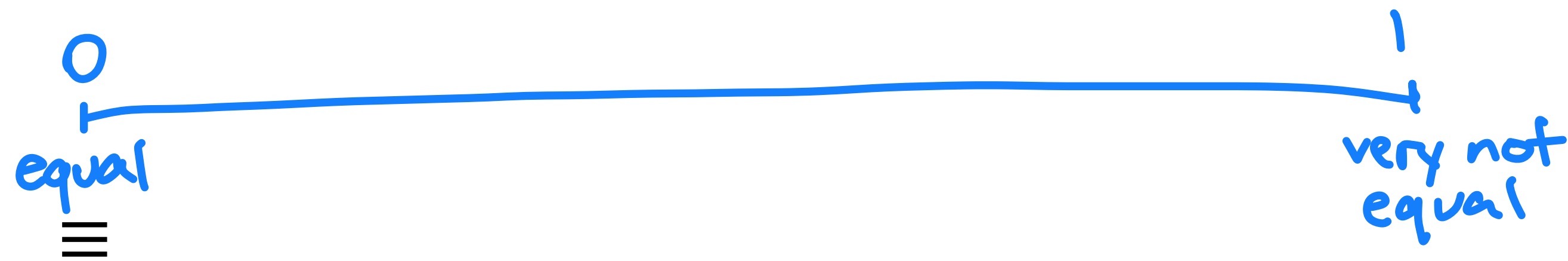
$$\Delta(U_C, U_{C'}) \leq \epsilon \iff C \equiv_{\epsilon} C'$$

Hilbert-Schmidt Distance

"easy" to compute

$$\Delta_{HS}(U, U') := \sqrt{1 - \frac{|\text{Tr}(U^\dagger U')|^2}{2^{2n}}}$$

2²ⁿ ← # qubits



Upper bound on total variation distance

Why Approximate Circuits?

FTQC **requires**
approximations for
arbitrary angle rotations

Ideal

Circuit C + No noise = U_C

Reality

Circuit C + Noise = $U_{C_{noisy}}$

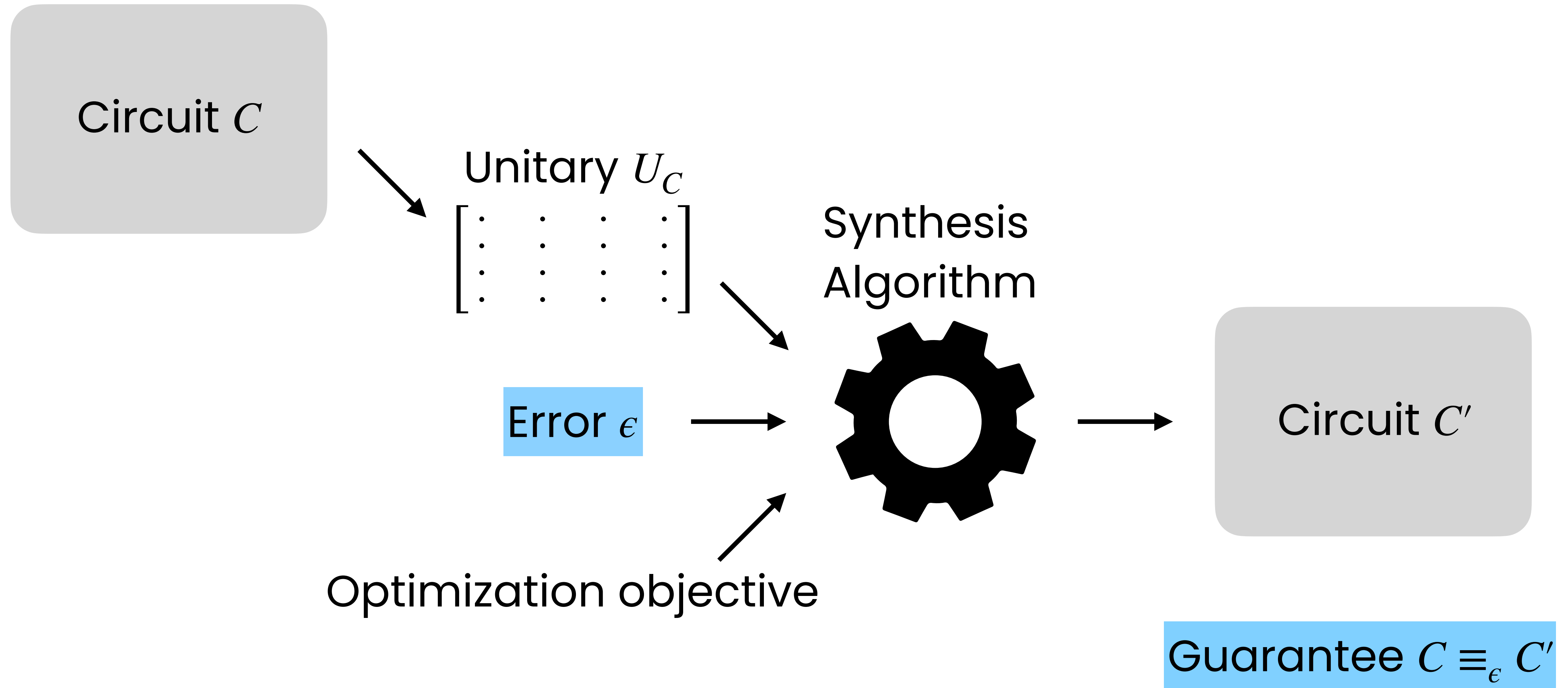
Circuit
 $C' \equiv_{\epsilon} C$

k fewer gates than C

The hope: $U_{C'_{noisy}}$ closer to U_C than $U_{C_{noisy}}$

Noise from k gates $> \epsilon$

Unitary Synthesis



Rich Research Area


Synthesis of Quantum-Logic Circuits

Vivek V. Shende, Stephen S. Bullock, and Igor L. Markov, *Senior Member, IEEE*

Towards Optimal Topology Aware Quantum Circuit Synthesis

Marc G. Davis, Ethan Smith, Ana Tudor, Koushik Sen, Irfan Siddiqi, Costin Iancu
Lawrence Berkeley National Laboratory

{marc.davis, ethan

 **bqskit** Public
Berkeley Quantum Synthesis Toolkit
● OpenQASM ☆ 132 📄 38

LEAP: Scaling Numerical Optimization Based Synthesis Using an Incremental Approach

ETHAN SMITH and MARC GRAU DAVIS, University of California, Berkeley
JEFFREY LARSON, Argonne National Laboratory

Exact synthesis of multiqubit Clifford+ T circuits

Brett Giles
Department of Computer Science
University of Calgary

Peter Selinger

RIJSEN, and COSTIN IANCU,

Modular Component-Based Quantum Circuit Synthesis

CHAN GU KANG, Korea University, Republic of Korea

Republic of Korea

SynthetiQ: Fast and Versatile Quantum Circuit Synthesis

ANOUK PARADIS*, ETH Zurich, Switzerland
JASPER DEKONINCK*, ETH Zurich, Switzerland
BENJAMIN BICHSEL, ETH Zurich, Switzerland
MARTIN VECHEV, ETH Zurich, Switzerland

 **synthetiQ** Public
● OpenQASM ☆ 3 📄 MIT

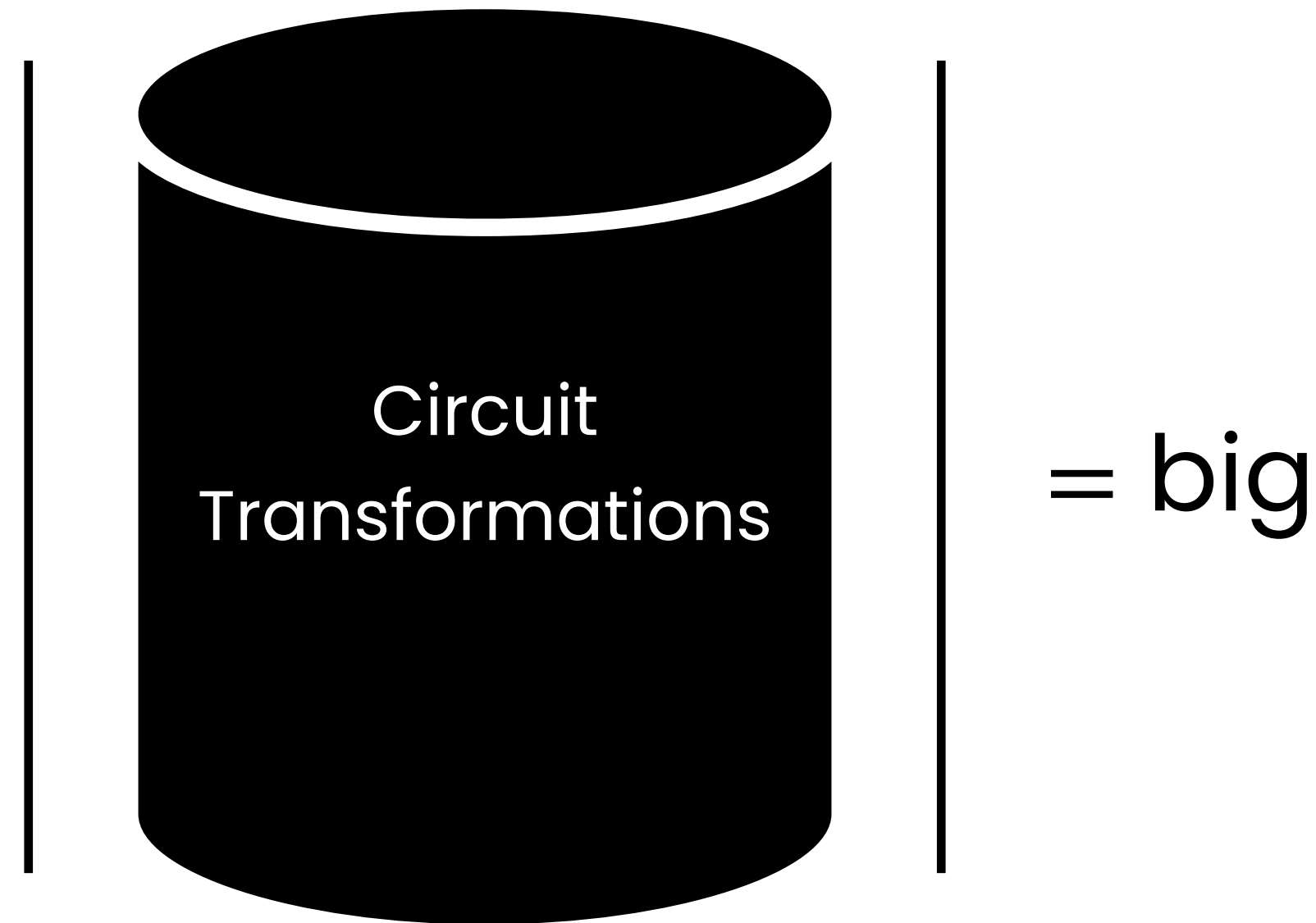
Techniques:

- analytical
- numerical optimization
- explicit search

Scheduling Transformations

Scheduling is hard

In what order to apply???

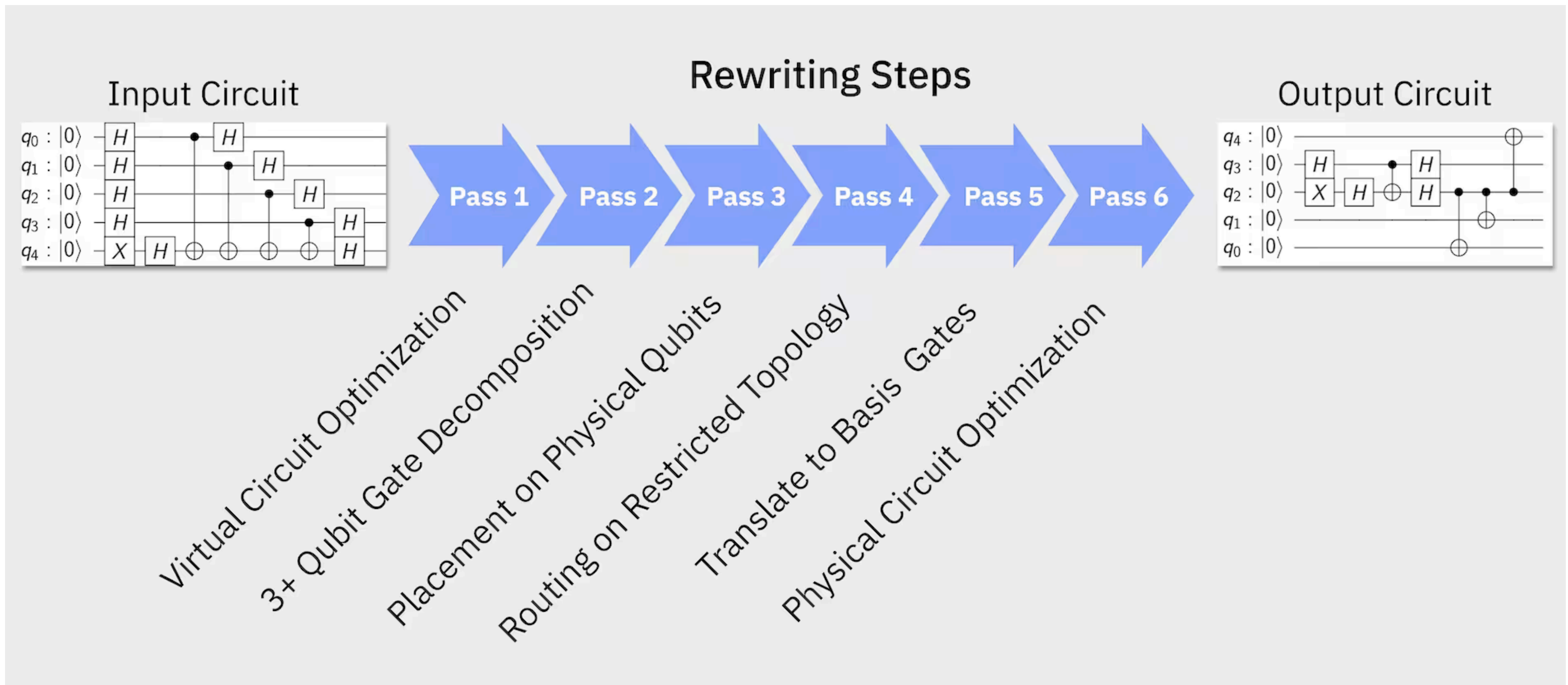


"phase-ordering problem"

Qiskit

<https://docs.quantum.ibm.com/api/qiskit/transpiler#optimization-stage>

https://github.com/Qiskit/qiskit/blob/stable/1.4/qiskit/transpiler/preset_passmanagers/level3.py



<https://docs.quantum.ibm.com/api/qiskit/transpiler#optimization-stage>

Qiskit

https://github.com/Qiskit/qiskit/blob/stable/1.4/qiskit/transpiler/preset_passmanagers/level3.py

qiskit / qiskit / transpiler / preset_passmanagers / level3.py

↑ Top

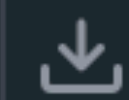
Code

Blame

119 lines (104 loc) · 4.8 KB · ⓘ



Raw



26

27 `def level_3_pass_manager(pass_manager_config: PassManagerConfig) -> StagedPassManager:`

28 `"""Level 3 pass manager: heavy optimization by noise adaptive qubit mapping and`
29 `gate cancellation using commutativity rules and unitary synthesis.`

30

31

This pass manager applies the user-given initial layout. If none is given, a search for a perfect layout (i.e. one that satisfies all 2-qubit interactions) is conducted. If no such layout is found, and device calibration information is available, the circuit is mapped to the qubits with best readouts and to CX gates with highest fidelit

The pass manager then tra the coupling constraints.
It is then unrolled to th **coarse fixed passes** rections are fixed.

Finally, optimizations in the form of commutative gate cancellation, resynthesis of two-qubit unitary blocks, and redundant reset removal are performed.

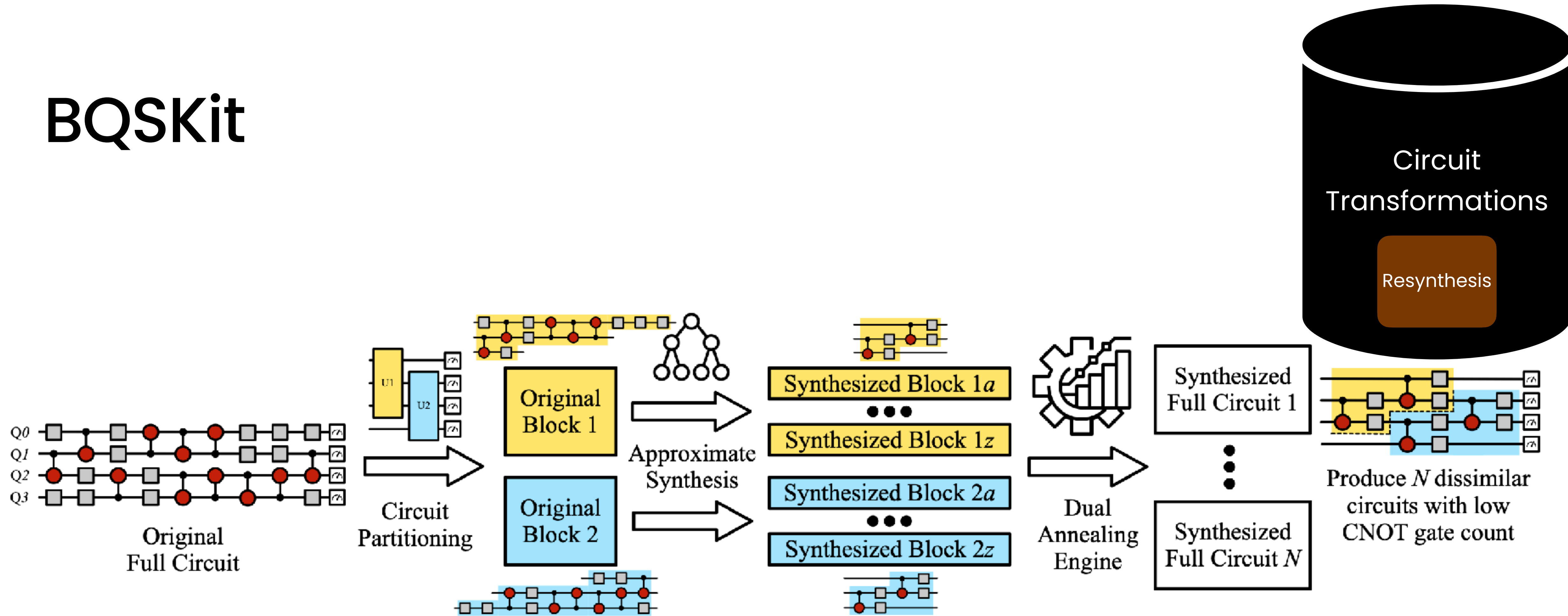
Circuit

Transformations

Rewrite
rules

Resynthesis

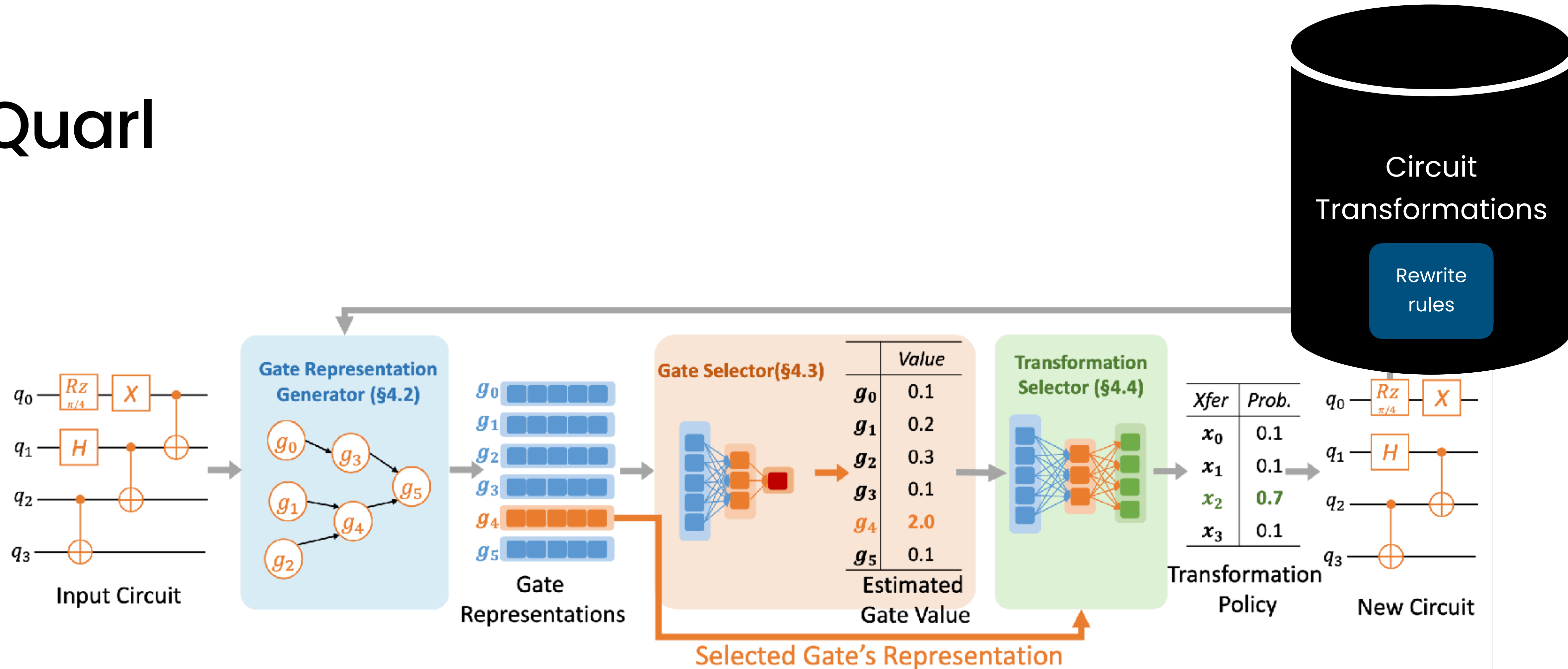
BQSKit



QUEST: systematically approximating quantum circuits for higher output fidelity [Patel et al. 2022]

<https://github.com/BQSKit/bqskit>

Quarl



* requires an NVIDIA A100 GPU (\$\$\$)

Quarl: A Learning-Based Quantum Circuit Optimizer [Li et al. 2024]

GUOQ

Optimizing Quantum Circuits, Fast and Slow

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Abstract

Optimizing quantum circuits is critical: the number of quantum operations needs to be minimized for a successful evaluation of a circuit on a quantum processor. In this paper we unify two disparate ideas for optimizing quantum circuits, *rewrite rules*, which are fast standard optimizer passes, and *unitary synthesis*, which is slow, requiring a search through the space of circuits. We present a clean, unifying framework for thinking of rewriting and resynthesis as abstract circuit transformations. We then present a radically simple algorithm, GUOQ, for optimizing quantum circuits that exploits the synergies of rewriting and resynthesis. Our extensive evaluation demonstrates the ability of GUOQ to strongly outperform existing optimizers on a wide range of benchmarks.

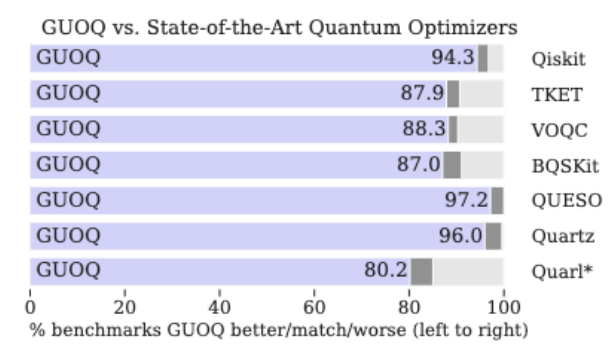
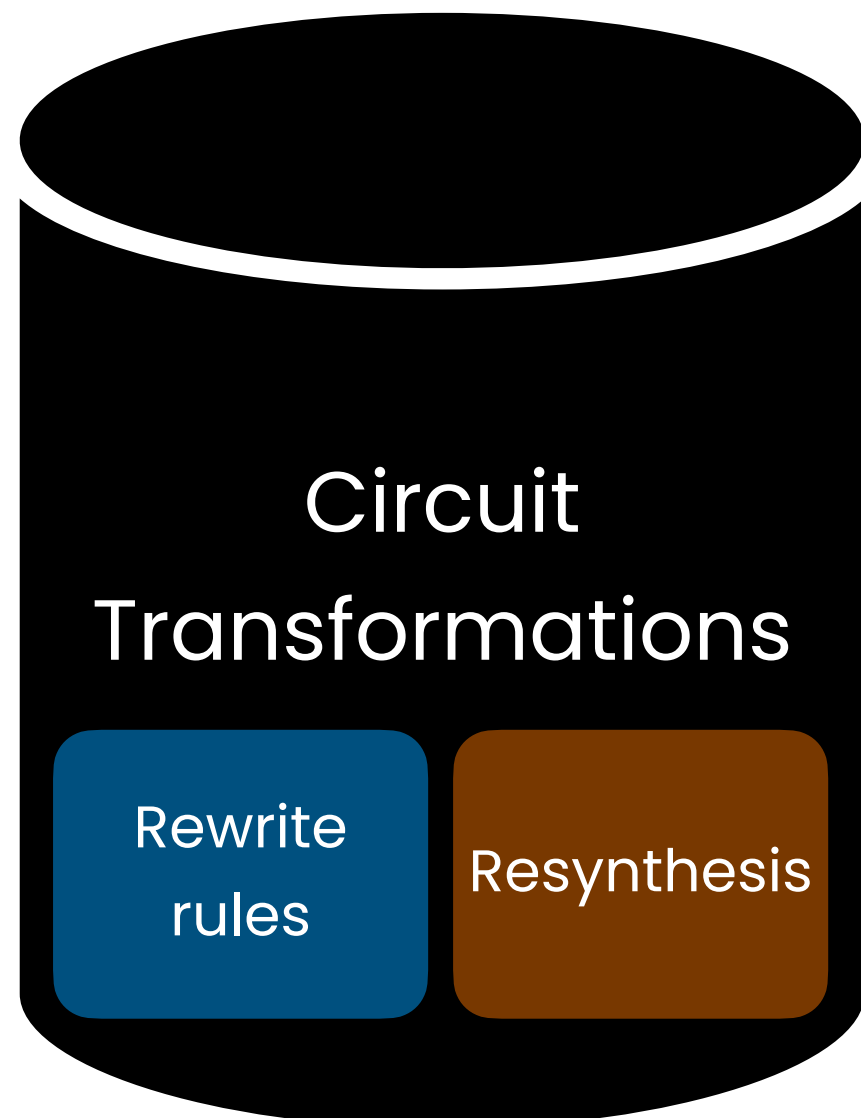


Figure 1. Summary of GUOQ compared to state-of-the-art on 2-qubit-gate reduction for the IBMQ20 gate set. GUOQ and BQSKit are allowed to approximate the circuit up to $\epsilon = 10^{-8}$. *Quarl requires an NVIDIA A100 (40GB) GPU to run.

ASPLOS talk on April 1st!



1. Randomly pick a transformation.
2. Randomly pick a subcircuit to transform.
3. Accept the result if better than current. Else, reject with high probability.

GUOQ

Optimizing Quantum Circuits, Fast and Slow

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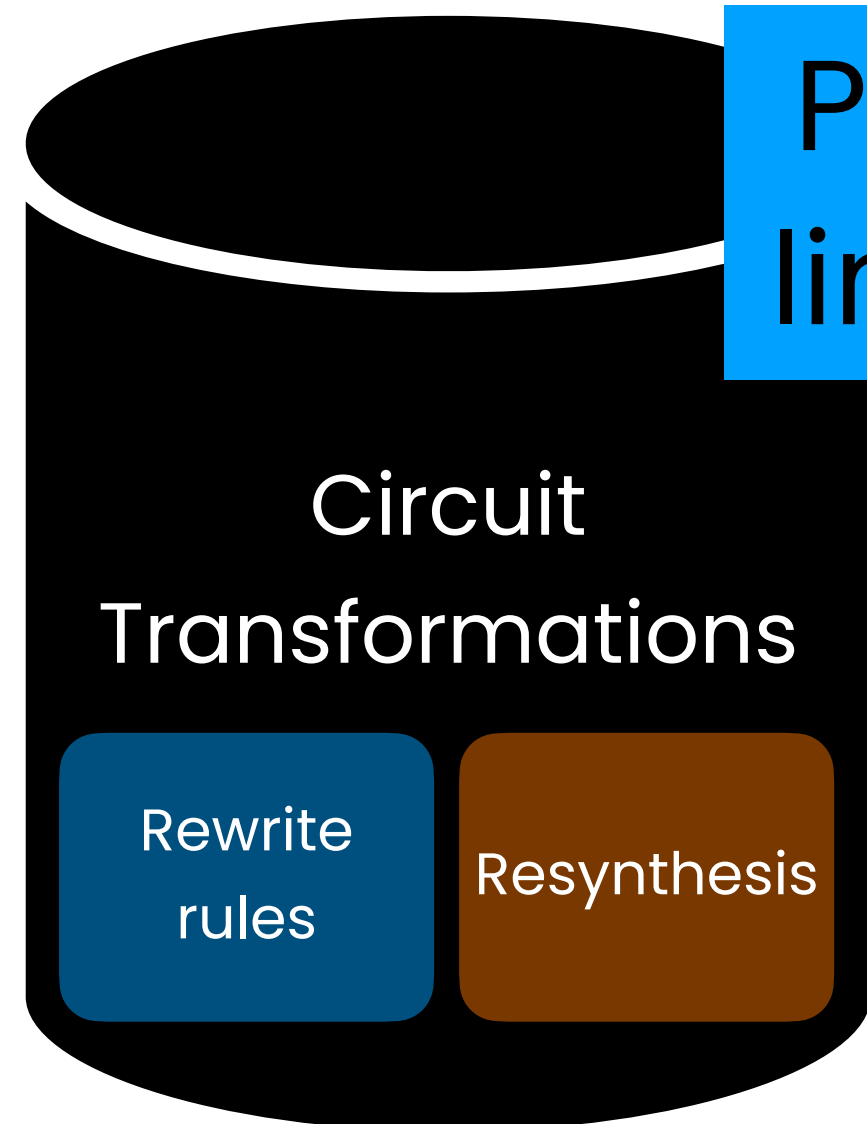
Aws Alharghouthi
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Abstract
Optimizing quantum circuits is critical: the number of quantum operations needs to be minimized for a successful evaluation of a circuit on a quantum processor. In this paper we unify two disparate ideas for optimizing quantum circuits, *rewrite rules*, which are fast standard optimizer passes, and *ancillary synthesis*, which is slow, requiring a search through the space of circuits. We present a clean, unifying framework for thinking of rewriting and resynthesis as abstract circuit transformations. We then present a radically simple algorithm, *GUOQ*, for optimizing quantum circuits that exploits the synergies of rewriting and resynthesis. Our extensive evaluation demonstrates the ability of *GUOQ* to strongly outperform existing optimizers on a wide range of benchmarks.

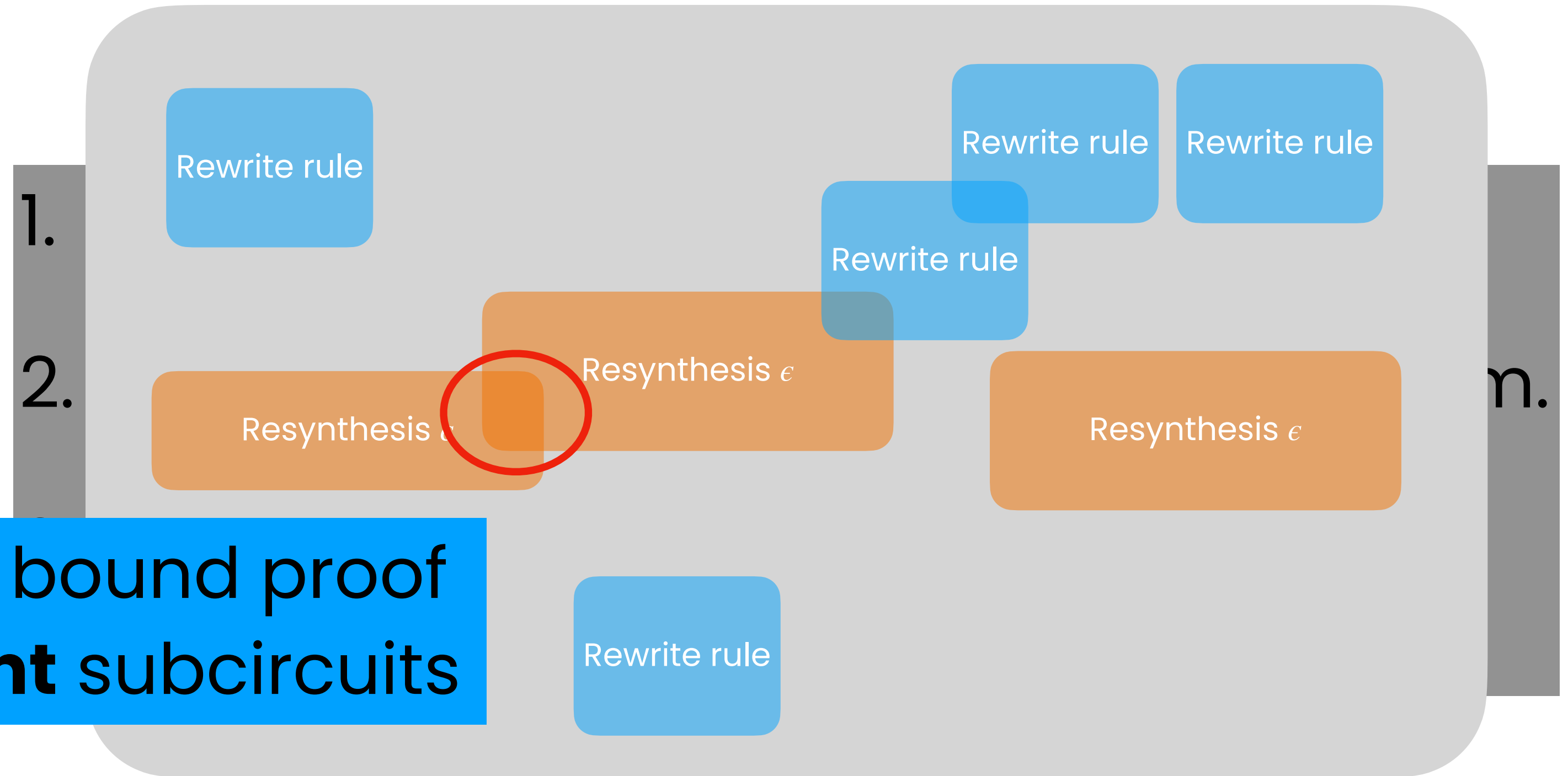
| GUOQ vs. State-of-the-Art (Quantum Optimizers) | GUOQ | State-of-the-Art |
|--|------|------------------|
| GUOQ | 94.3 | Qiskit |
| GUOQ | 87.9 | TKET |
| GUOQ | 88.3 | VUQC |
| GUOQ | 87.0 | QOSKit |
| GUOQ | 87.2 | QIPRO |
| GUOQ | 86.0 | Quarta |
| GUOQ | 84.2 | Quartus |

Figure 1. Summary of *GUOQ* compared to state-of-the-art on 2-qubit gate reduction for the *uqcq* gate set. *GUOQ* and *QOSKit* are allowed to approximate the circuit up to $\epsilon = 10^{-6}$. *Quartus requires an NVIDIA A100 (80GB) GPU to run.

ASPLOS talk on April 1st!



Prior work: error bound proof limited to **disjoint** subcircuits

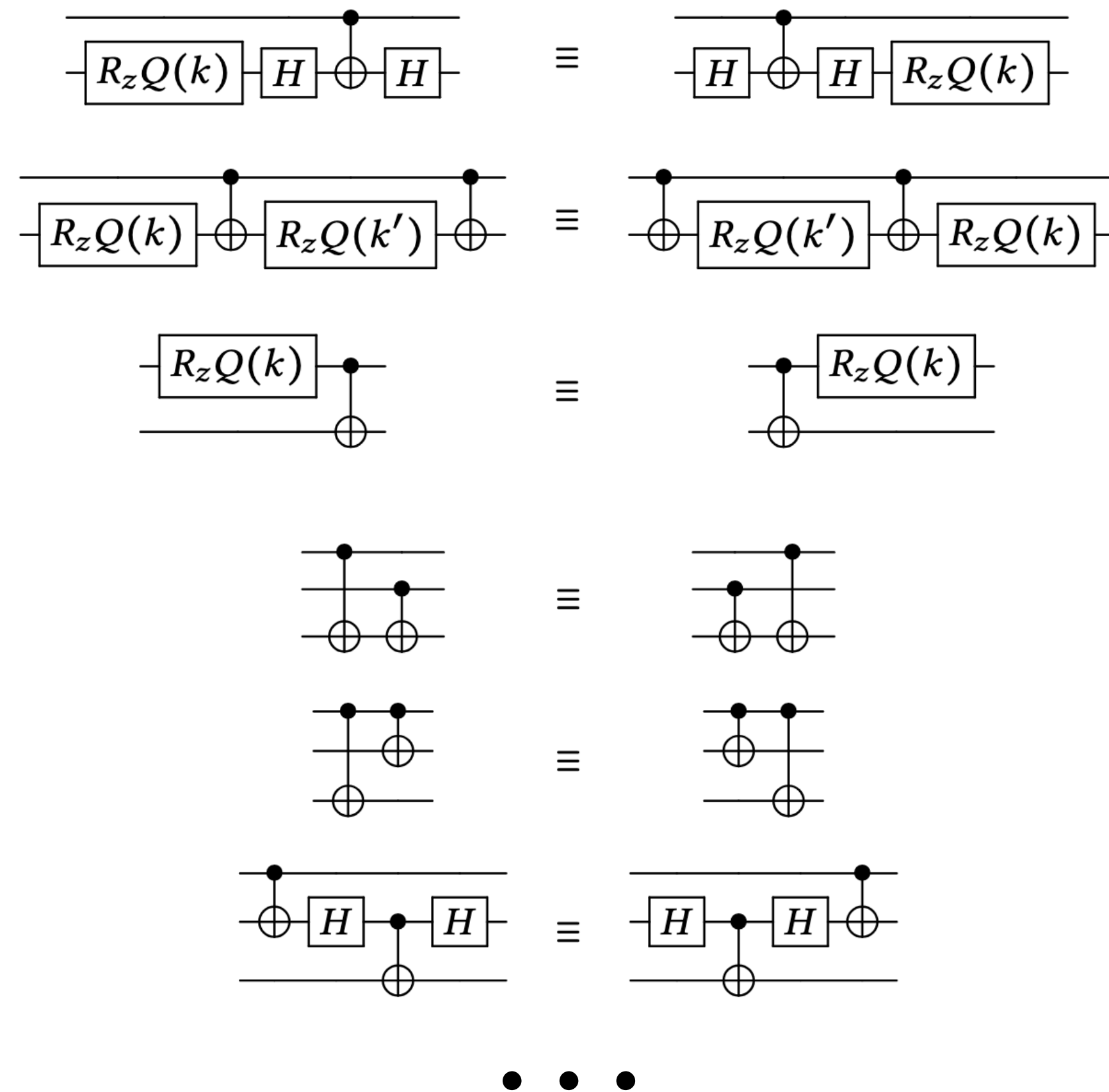


$$\epsilon_{total} \leq \epsilon + \epsilon + \epsilon$$

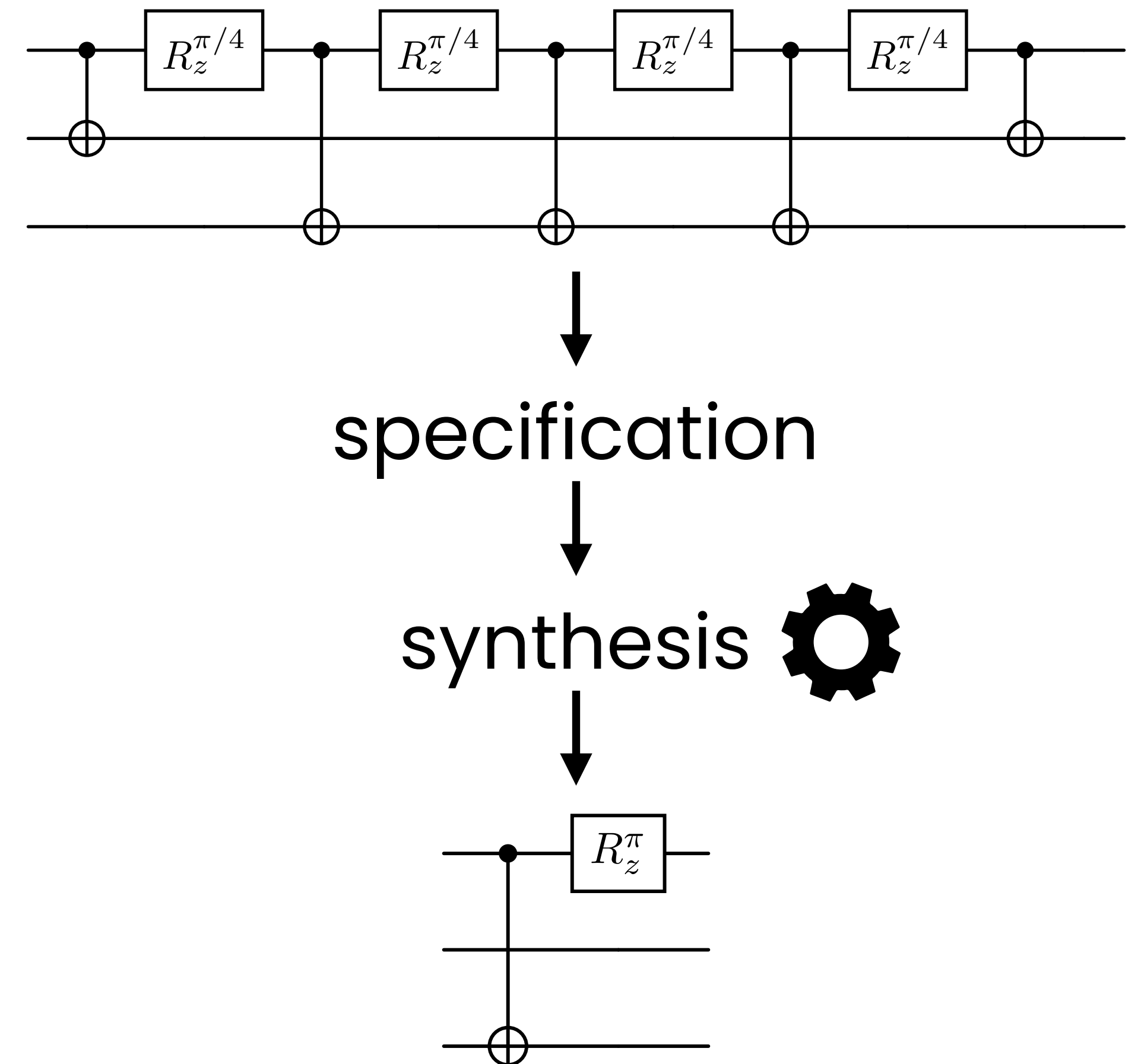
We extended to handle arbitrary subcircuits

GUOQ Motivation: Two Disparate Techniques

Rewrite Rules

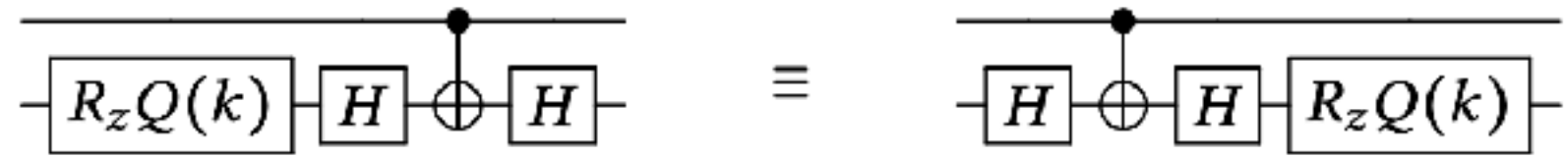


Circuit Resynthesis



GUOQ Motivation: Two Disparate Techniques

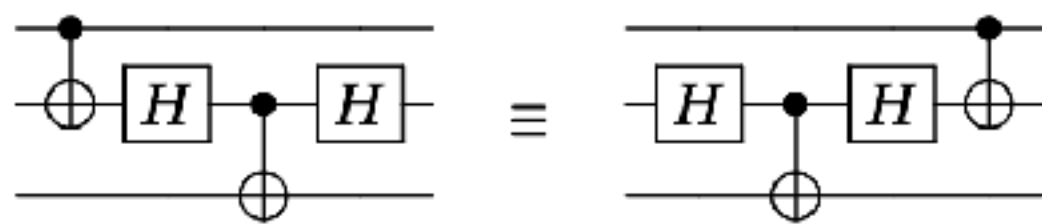
Rewrite Rules



Fast

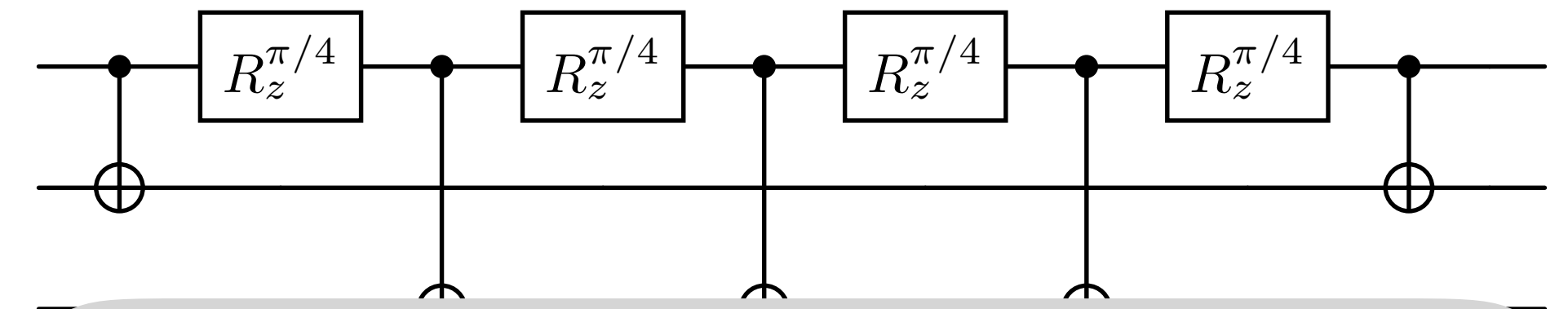
Limited by # gates

Exact



...

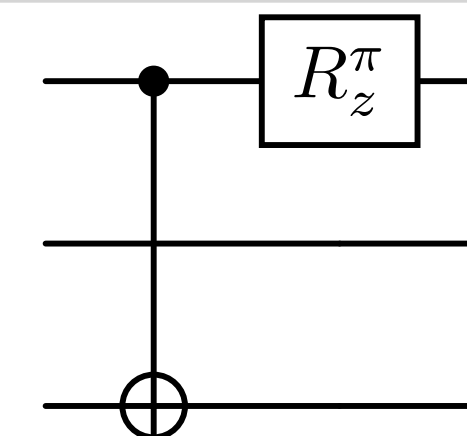
Circuit Resynthesis



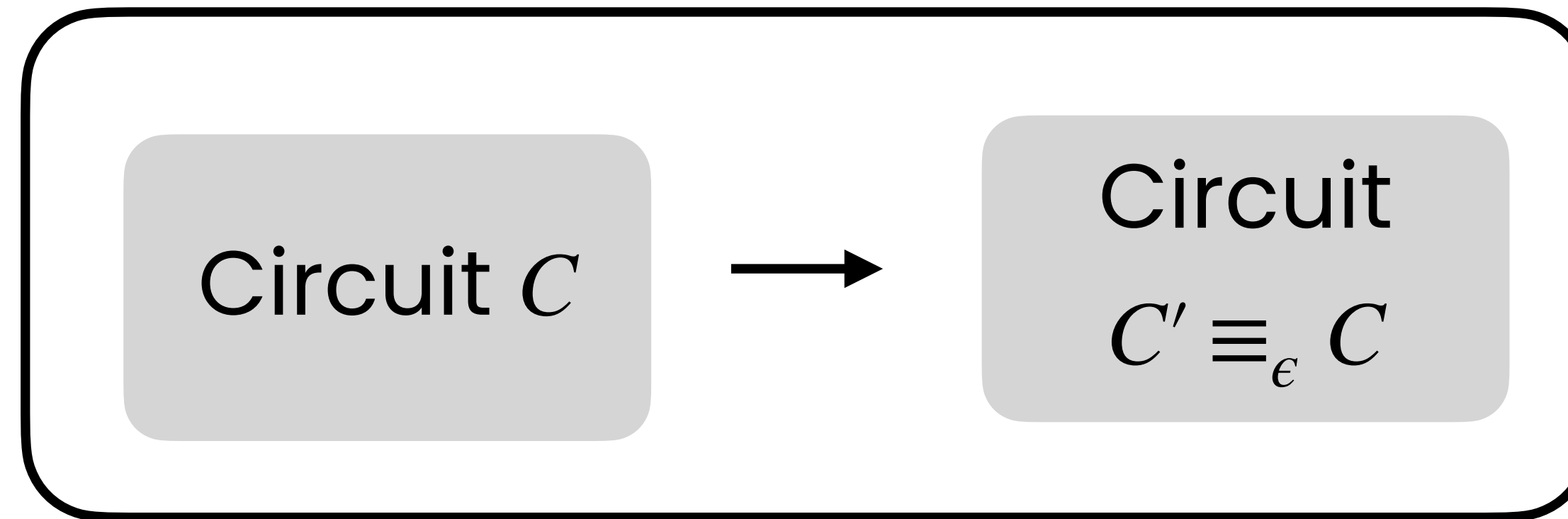
Slow

Limited by # qubits

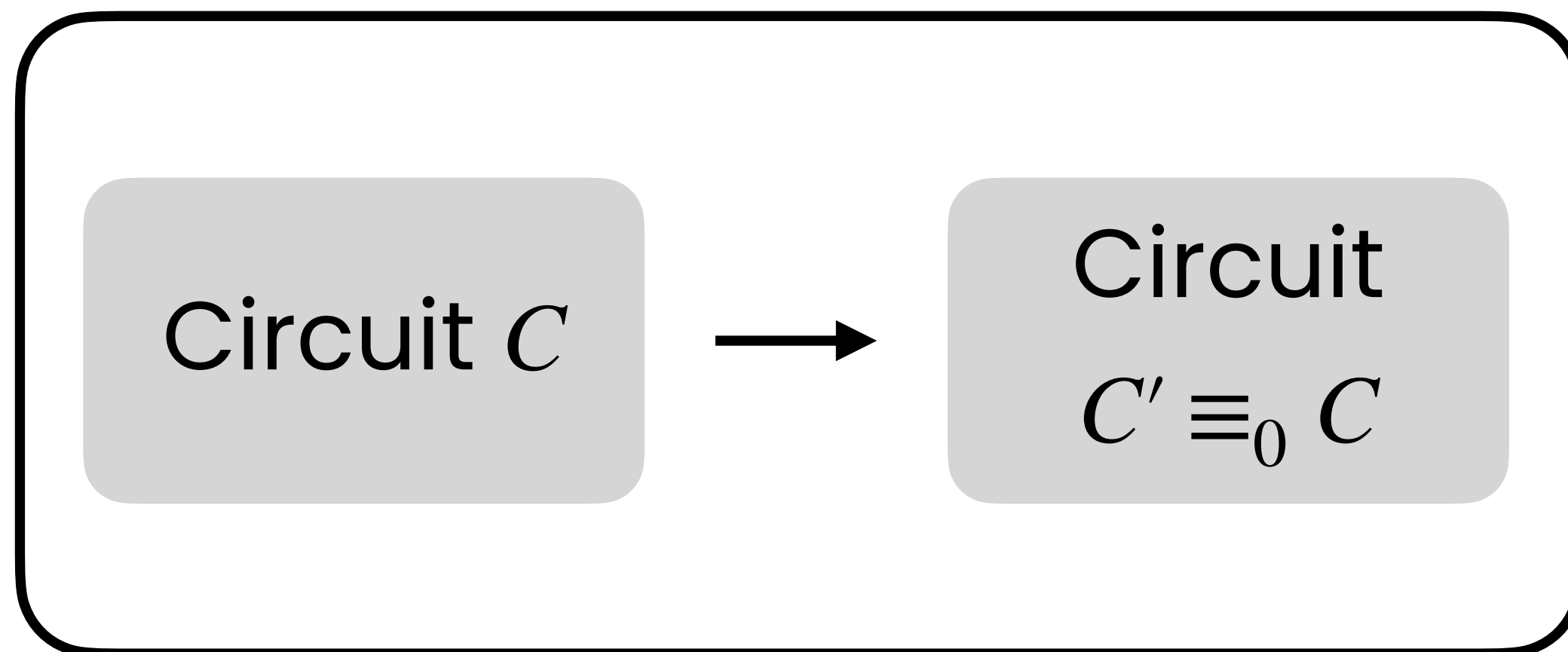
Approximate



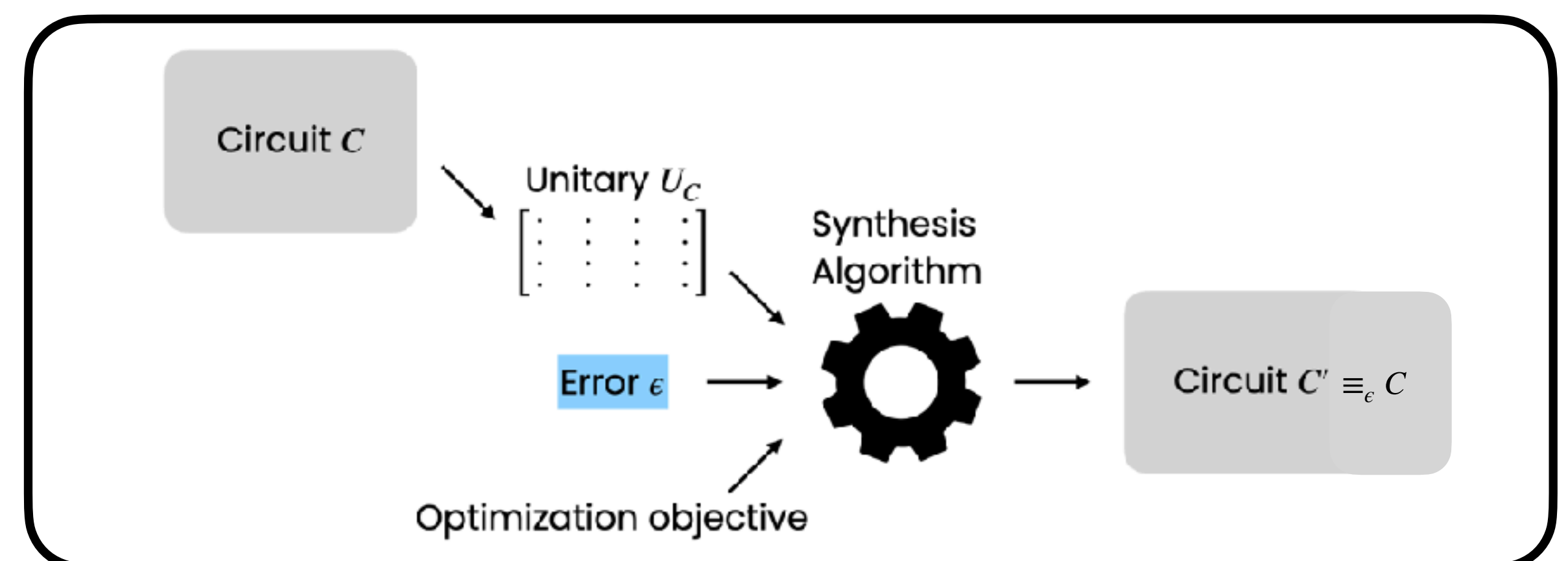
Unifying Abstract Circuit Transformations



transformation $\tau_{\epsilon} : \mathcal{C} \rightarrow \mathcal{C}$

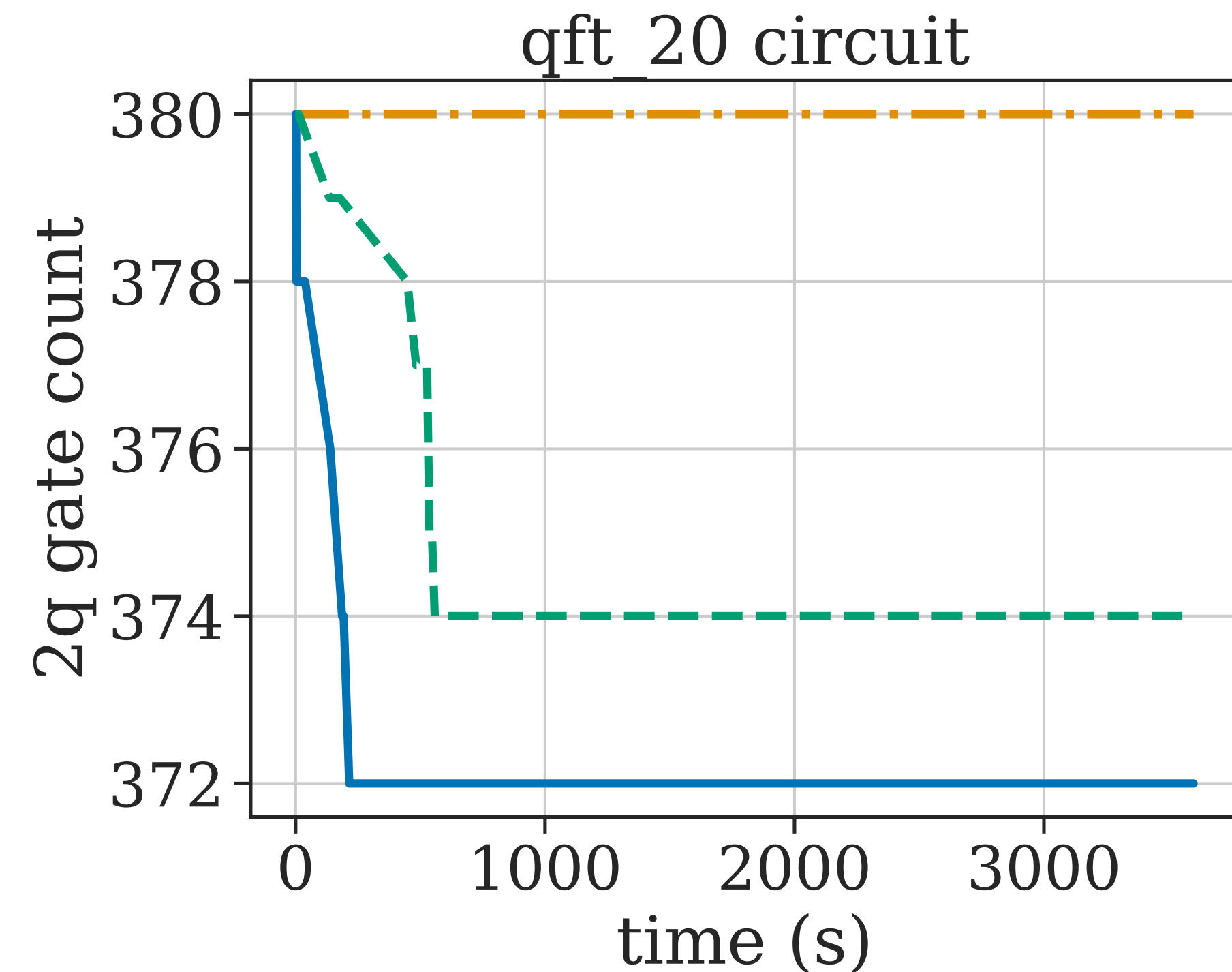
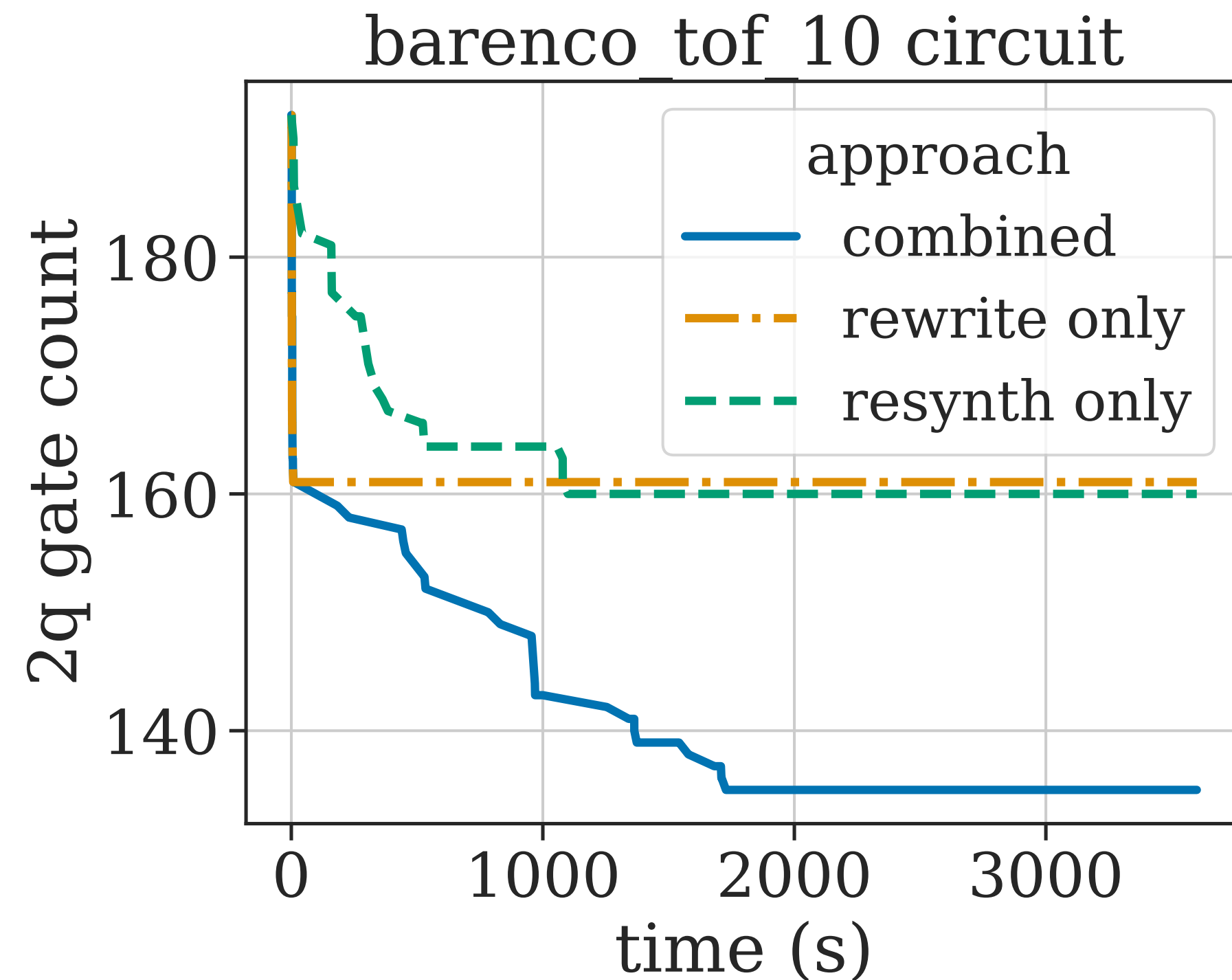


rewrite rule



resynthesis

GUOQ Search Exploits Synergy



Also significantly outperforms state-of-the-art!

Come to the ASPLOS talk on April 1st!

Optimizing for FTQC

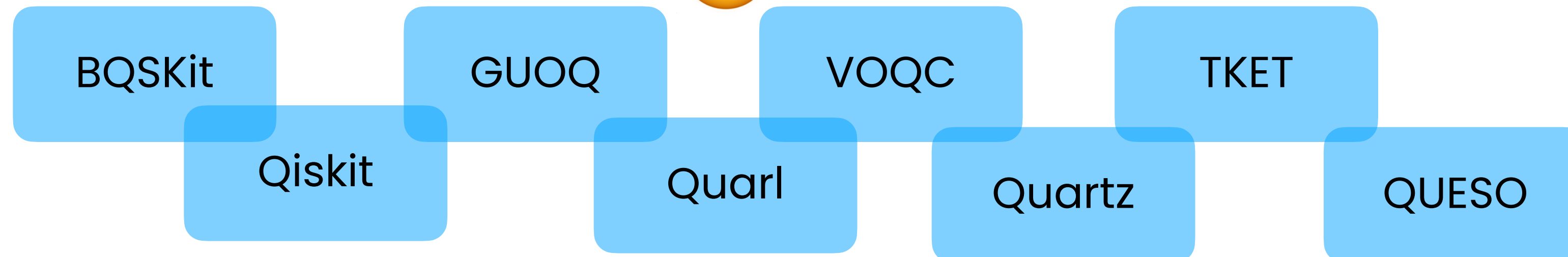
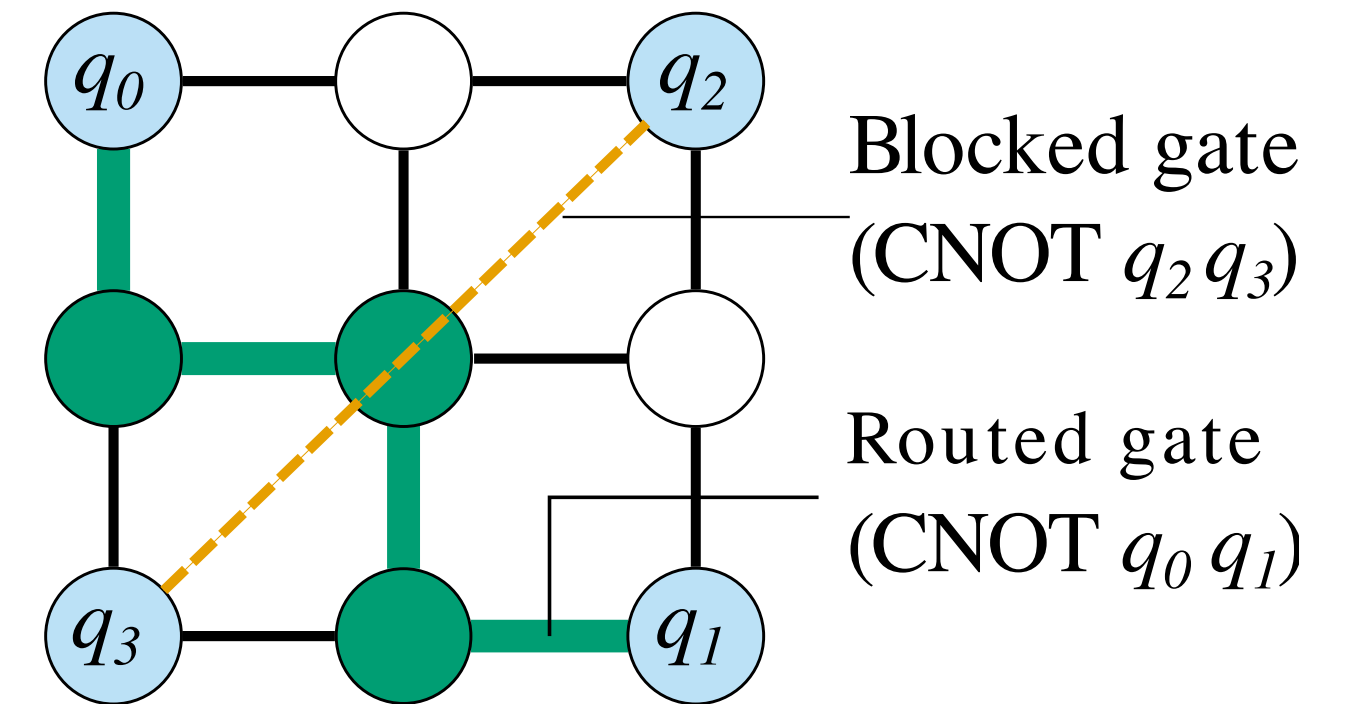
Optimizing for FTQC

Clifford + T gate set: $\{T, T^\dagger, S, S^\dagger, H, CX\}$

discrete

"magic state distillation"

Eastin-Knill theorem: no transversal universal set



Optimizing for FTQC

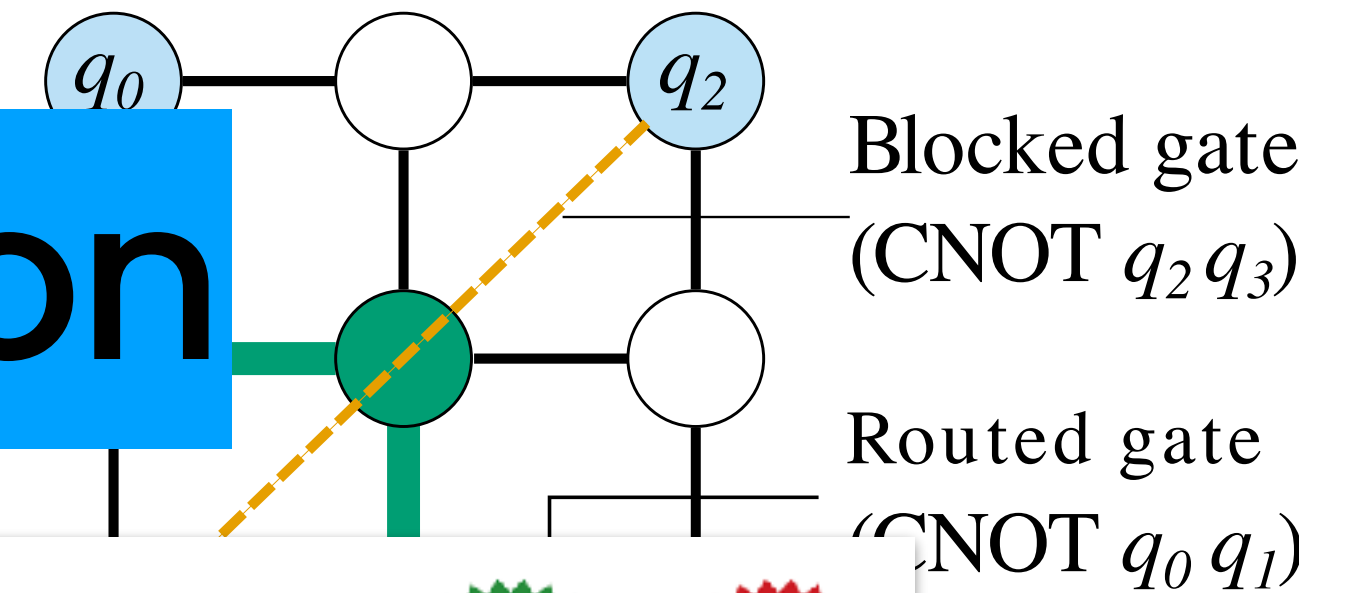
Clifford + T gate set: $\{T, T^\dagger, S, S^\dagger, H, CX\}$

discrete

"magic state distillation"

Eastin-Knill th

Open Research Question



Reducing T-count with the ZX-calculus

Aleks Kissinger and John van de Wetering

Radboud University Nijmegen
January 20, 2020

Linear and Non-linear Relational Analyses for Quantum Program Optimization

MATTHEW AMY, Simon Fraser University, Canada
JOSEPH LUNDERVILLE, Simon Fraser University, Canada



BQSKIT

GUOQQ

VOQQC

TKET

Qiskit

Quarl

Quartz

QUESO